CS 173 Lecture 8c: Graph Isomorphism

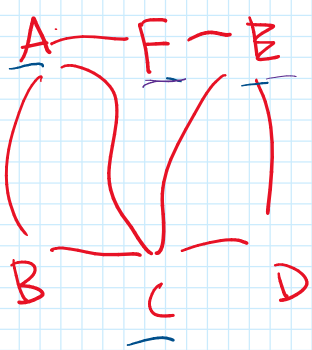
Graphs capture connectivity properties

Two graphs G & H are isomorphic ($G \cong H$) if (informally) they have the same connectivity structure.

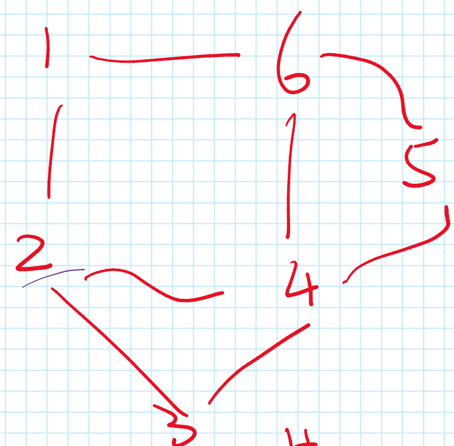
i.e., there is a way to line the vertices of H up w/ the vertices of G such that all edges match.

i.e., if $G = (V, E)$ & $H = (V', E')$, there is a bijection $f: V \to V'$ s.t. $\underline{\{u,v\} \in E \Leftrightarrow \{f(u), f(v)\} \in E'}$.

onto & one-to-one



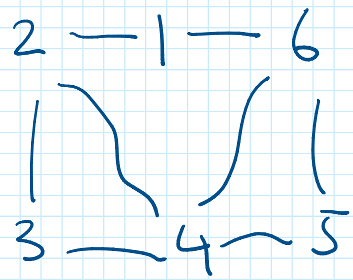
G



H

e.g.

v	f(v)	g(v)
A	2	6
B	3	5
C	4	4
D	5	3
E	6	2
F	1	1



H

consequences of isomorphism:

$$\deg_G(v) = \deg_H(f(v))$$

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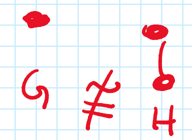
let $v_1, v_2, \ldots, v_n$ be a walk in G.
Then $f(v_1), f(v_2), \ldots f(v_n)$ is a walk in H.
cycle, path

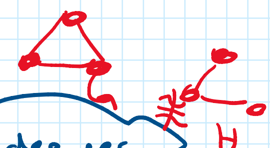
Proving non-isomorphism between $G=(V,E)$ & $H=(V',E')$

1). If $|V| \neq |V'|$ then $G \ncong H$

if $f: A \to B$ is a bijection $|A| = |B|$

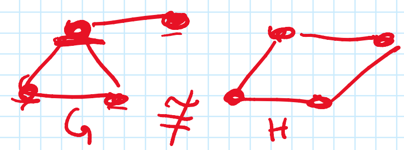


2.) If $|E| \neq |E'|$ then $G \ncong H$



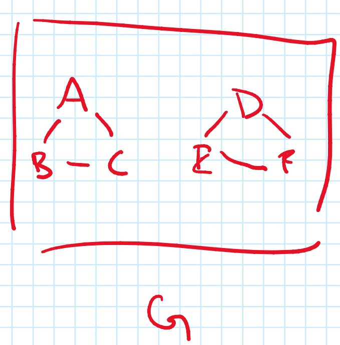
list of vertex degrees in descending order

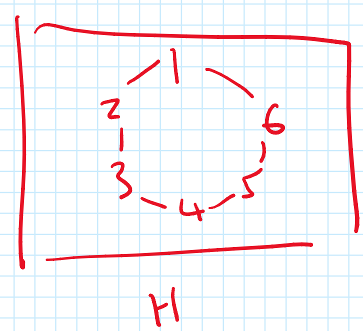
3.) If degree sequences do not match, then $G \ncong H$.



$3,2,2,1 \neq 2,2,2,2.$

4.) If other properties don't line up, then $G \ncong H$





$|V| = |V'|$

$|E| = |E'|$

degree sequences: $2,2,2,2,2,2 = 2,2,2,2,2,2.$

A, B, C is a cycle in G.

But there is no cycle of length 3 in H.

$G \ncong H$

But there is no cycle of length 3 in H.

$$G \not\cong H$$

Cycles, paths, walks, subgraphs

5. Brute Force Search