CS 173 Lecture 6a: Relations

Definition. A relation on a set A is subset $R \subseteq A \times A$.
if $(a, b) \in R$, a is related to b, or aRb for short
($a \in A$, $b \in A$)

Ex. $A = \mathbb{R}$ $R = \{(a,b) \in \mathbb{R} \times \mathbb{R} : a < b\}$ "$<$"
$R =$ "$<$"

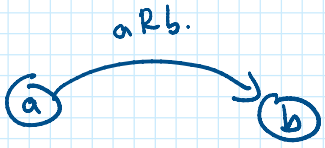
$A = \mathbb{R}$ $R = \{(a,b) \in \mathbb{R} \times \mathbb{R} : a \leq b\}$ "$\leq$"

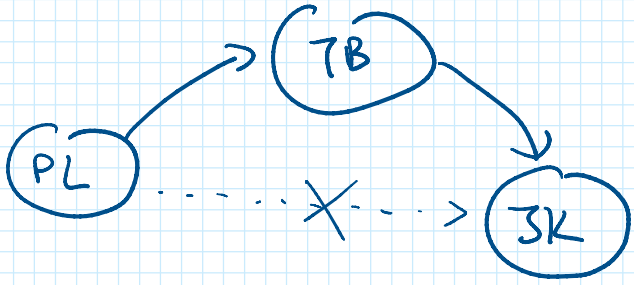
$A = \mathbb{R}$ $R = \{(a,b) \in \mathbb{R} \times \mathbb{R} : a = b\}$ "$=$"

$A = \mathbb{Z}$ $R = \{(a,b) \in \mathbb{R} \times \mathbb{R} : a \mid b\}$ "$\mid$"
$R =$ "$\mid$"

A = set of Instagram users "follows"
$R = \{(a,b) \in A \times A : a \text{ follows } b\}$

A = set of courses at UIUC "prereq"
$R = \{(a,b) \in A \times A : a \text{ is a pre-req for } b\}$





Definitions:

A relation R on A

- reflexive if aRa for all $a \in A$.
  $<$ ✗ $\leq$ ✓ $=$ ✓ $\mid$ ✓
  negation: $\exists a \in A$ s.t. $a \not R a$
- irreflexive if $a \not R a$ for all $a \in A$
  negation: $\exists a \in A$ s.t. aRa

not negations of each other

$<$ ✓ $<$ ✗ $=$ ✗ $\mid$ ✗

$<$ ✓ $\quad \leq$ ✗ $\quad =$ ✗ $\quad \mid$ ✗

Suppose $A = \emptyset$. Consider any relation R on A

for all $a \in A$, is aRa? ✓
is $a \not R a$? ✓

Suppose $A = \mathbb{R}$.

$R = \{(a,b) \in A \times A : ab = 1\}$.

for all $a \in A$, is aRa? ✗ $\quad 2 \not R 2$
is $a \not R a$? ✗ $\quad 1 R 1$

not negations

- symmetric if for all $a, b \in A$, $aRb \to bRa$.

$<$ ✗ $\quad \leq$ ✗ $\quad =$ ✓ $\quad \mid$ ✗ $\quad$ follows ✓

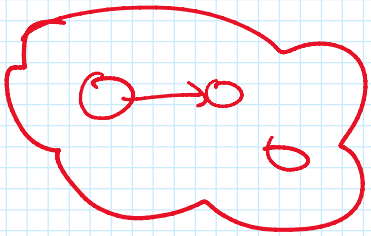
facebook friends ✓

- antisymmetric if for all $a, b \in A$, aRb & $bRa \to a = b$.

$<$ ✓ $\quad \leq$ ✓ $\quad =$ ✓ $\quad \mid$ ✓ $\quad$ follows ✗

- transitive: if for all $a, b, c \in A$, aRb & $bRc \to aRc$

$<$ ✓ $\quad \leq$ ✓ $\quad =$ ✓ $\quad \mid$ ✓ $\quad$ follows ✗



transitive