CS 173 Lecture 2c: More Proofs

Claim: For all real numbers a, b, n, if $ab \le n$, then $a \le \sqrt{n}$ or $b \le \sqrt{n}$.

Proof by contrapositive: for all real numbers a, b, n, if $a > \sqrt{n}$ and $b > \sqrt{n}$ then $ab > n$.

Suppose that $a > \sqrt{n}$ and $b > \sqrt{n}$.

Then $ab > \sqrt{n}\sqrt{n} = n$. $\square$

Claim: For all integers n, $n^2 \ge n$.

Proof. Let n be an integer.

First suppose $n \ge 1$. Then $\underbrace{n^2 \ge n \cdot 1}_{n \ge 1} = n$.

Next, suppose $n = 0$. Then $n^2 = 0 \cdot 0 = 0 = n$.

Finally, suppose $n \le -1$. $n^2 = (-n)(-n) \ge 0 > -1 \ge n$.

In all cases, $n^2 \ge n$, completing the proof. $\square$

Claim: (AM-GM Inequality)

For all real numbers a, b, such that $ab \ge 0$: $\underbrace{\frac{a+b}{2}}_{\text{arithmetic mean}} \ge \underbrace{\sqrt{ab}}_{\text{geometric mean.}}$

Scratchwork: $\frac{a+b}{2} \ge \sqrt{ab}$.

Square: $\frac{(a+b)^2}{4} \ge ab$

Simplify: $\frac{a^2 + 2ab + b^2}{4} \ge ab$

rearrange: $a^2 - 2ab + b^2 \ge 0$

$\frac{a+b}{2} \ge \sqrt{ab} \rightarrow \frac{(a+b)^2}{4} \ge ab$,

$p \rightarrow q$

$\not\equiv q \rightarrow p$. !!

$p \leftrightarrow q$

$\equiv$

rearrange: $$\frac{a^2-2ab+b^2}{4} \geq 0 \qquad \qquad p \leftrightarrow q \equiv q \leftrightarrow p.$$

simplify: $$(a-b)^2 \geq 0.$$

Proof. Observe $(a-b)^2 \geq 0$. Then $a^2-2ab+b^2 \geq 0$, which implies that $\frac{a^2-2ab+b^2}{4} \geq 0$. Adding ab to both sides, we get $\frac{a^2+2ab+b^2}{4} \geq ab$. Since $ab \geq 0$, take square roots of both sides to get $\frac{(a+b)^2}{2} \geq \sqrt{ab}$, as desired. $\square$