CS 173 Lecture 1a: Propositional Logic & Predicate Logic

This video: define propositions & predicates

- Proposition: statement that is TRUE/FALSE   Boolean

  not a question/command   no variable

  2+2=4   TRUE

  Patrick is the Queen of England   FALSE

  Pineapple is a valid pizza topping   ?? subjective

  over there   preposition

  2+2   number

- Combining Proposition

  Let p & q be propositions

NOT ($\neg$) $\neg p$ is TRUE when p is FALSE

AND ($\wedge$) $p \wedge q$ is TRUE when p is TRUE & q is TRUE

OR ($\vee$) $p \vee q$ is TRUE when at least one of p & q is TRUE

exclusive   inclusive

XOR ($\oplus$) $p \oplus q$ is TRUE when exactly one of p & q is TRUE

IMPLICATION ($\rightarrow$) $p \rightarrow q$ is TRUE when (p is TRUE & q is TRUE) or (p is FALSE)

"p implies q"

"if p then q"

If $\underbrace{\text{this statement is written in purple,}}_{p}$ then $\underbrace{\text{the sky is green}}_{q}$

TRUE until proven FALSE

IFF ($\leftrightarrow$) $p \leftrightarrow q$ is TRUE when $p \rightarrow q$ & $q \rightarrow p$

$\underbrace{\text{The sky is green}}_{p}$ if and only if $\underbrace{\text{pigs can fly.}}_{q}$

Truth Table shows possible values complex positions

p q

Truth Table shows possible values complex prositions

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\neg p \vee q \equiv p \to q$

- Predicate statement w/ variable that becomes a proposition when values are substituted in for the variables

$x + 2 = 4$ predicate

- Quantifiers

universal quantifier $\forall$

eg. For all integers x, $x+2=4$. FALSE

$\forall x \in \mathbb{Z}, x+2=4$

There exists an integer x, such that $x+2=4$. TRUE

existential quantifier $\exists$

There exists a real number x, such that $x^2=2$ TRUE $x=\sqrt{2}$

$\exists x \in \mathbb{R}, x^2=2$

There exists an integer x, such that $x^2=2$

$\exists x \in \mathbb{Z}, x^2=2$ FALSE

$\mathbb{Z}$ integers

$\mathbb{Q}$ rational

$\mathbb{R}$ real numbers

this is how we write

For all integers x & y, if $x=y$ then $x-y=0$.

$\forall x, y \in \mathbb{Z}, \quad x=y \to x-y=0.$

this is shorthand (scratchwork / board)