CS 173 Lecture 16b: O calculations

$f(n) = \underbrace{O(g(n))}_{\text{kind of like } \leq}$ iff $\exists\, c, k \in \mathbb{R}$ s.t. $\forall n \geq k$,
$f(n) \leq c\,g(n)$.

$f(n) = \Theta(g(n))$ iff $f(n) = O(g(n))$ & $g(n) = O(f(n))$.

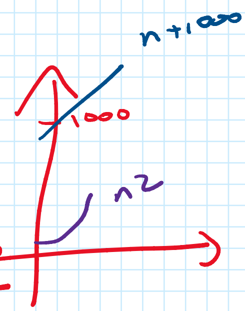
n vs n^2

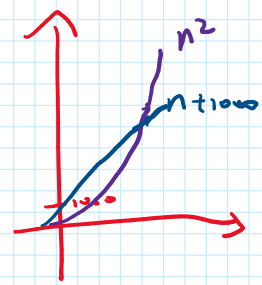
What we know: for $n \geq 1$,
$n = n \cdot 1 \leq n \cdot n \leq n^2$.
So: we can choose $c = k = 1$.

could have been less careful
in general: many possible choices.

$n + 1000$ vs. n^2.

is $n + 1000 = O(n^2)$?





$c = 1001,\ k = 1$	$c = 1,\ k = 1000$	$c = 1001,\ k = 1000$
for $n \geq 1$	for $n \geq 1000$	for $n \geq 1000$
$n + 1000$ $= n + 1000 \cdot 1$ $\leq n + 1000 \cdot n$ $= 1001\, n$ $= 1001\, n \cdot 1$ $\leq 1001\, n^2$	$n + 1000$ $\leq n + n$ $= 2n$ $\leq n \cdot n$ $= n^2$ $n \geq 1000 \geq 2$	$n + 1000$ $\leq n + n$ $\leq 2n$ $\leq 1001\, n$ $\leq 1001\, n^2$

kind of silly
but it works!

Yes: three different "proofs"

Useful technique: leading term

$$1 \ll \log n \ll (\log n)^2 \ll \cdots \ll n \ll n\log n \ll n(\log n)^2 \ll \cdots \ll n^2 \ll \cdots$$

$$\ll n^2 \log n \ll \cdots \ll n^3 \ll \cdots \ll 2^n \ll 3^n \ll \cdots \ll n! \ll n^n$$

Base of log doesn't matter:

$$\log_a n = \Theta(\log_b n) \quad \forall\, a, b > 0.$$

$$\left(\log_a n = \underbrace{\log_a b}_{\text{constant.}} \cdot \log_b n\right)$$

$$f(n) = \sum a_i\,(\text{term}_i)$$

$$\rightarrow f(n) = O(\text{leading term}).$$

$$n^2 + 3000 = O(n^2) = O(2^n)$$

$n^2 + 3000 = \Theta(n^2)$ "best" upper bound

an upper bound, not the best upper bound

(!!) leading term "shortcut" can be misleading.

$\log_{10} n + 10$ vs $n \log n$

{base doesn't matter}

Suppose I try "blindly" to replicate

$$n + 1000 \leq 1001\, n \leq 1001\, n^2$$

e.g. $\log n + 10 \le 11 \log n \le 11 n \log n$

$\longrightarrow$ can set $c = 11$, $k = 1$.

when $n = 1$,
$\log n = 0. \to \log n + 10 = 10,$
$n \log n = 0$

moral: do be careful about c's and k's.

c's & k's are <u>never</u> too big

can be too small.

in this case: better to do:

$$\log_b n + 10 \quad \text{vs} \quad n \log_b n$$

$$\log_b n + 10 \le 2 \log_b n \le 2 n \log_b n$$

for $n \ge ?$

need: $10 \le \log_b n$

$\Leftrightarrow b^{10} \le n$

for $n \ge b^{10}$