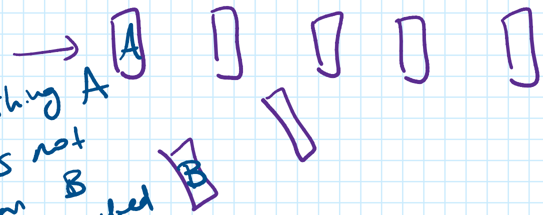
CS 173 Lecture 10c: All Your Base Cases

Dominos



pushing A does not mean B gets knocked over.

Part A:

Claim: $\forall n \geq 18\ \exists k, l$ $n = 3k + 10l$

Proof:

Case $n = 18$ . . .

$n = 19$ . . .

$n = 20$ . . .

Multiple Bases Cases Needed.

Moral of Today:

Can you have too many base cases? **NO!**

too few base cases? **YES!**

Fake Claim: $\forall n \in \mathbb{N},\quad 3n = 0.$

Fake Proof: By induction.

Base Case $n = 0$: $3n = 0$.

Inductive Hypothesis: Assume that for $0 \leq k < n$, $3k = 0$.

Inductive Step: Case $n > 0$.

$n = a + b$ where $3a = 0$ & $3b = 0$ since $0 \leq a, b < n$.

So $3n = 3(a+b) = 3a + 3b = 0 + 0 = 0.$ $\square$

$n = 1$, what are a, b?

What went wrong?

When $n = 1$, this inductive step cannot apply.

But $n = 1 \rightarrow 3n = 3$.

Fake Claim: $\forall n \in \mathbb{N},\quad \frac{d}{dx} x^n = 0.$

Fake Proof: By induction:

Base Case: $n = 0$. $\frac{d}{dx} x^0 = \frac{d}{dx} 1 = 0.$

I.H. For $0 \leq k < n$, $\frac{d}{dx} x^k = 0.$

Base case: $n=0$. $\frac{d}{dx} x^0 = \frac{d}{dx} 1 = 0$.

I.H. For $0 \le k < n$, $\frac{d}{dx} x^k = 0$.

I.S. $\frac{d}{dx} x^n = \frac{d}{dx}(x^1 \cdot x^{n-1}) = x\underbrace{\left(\frac{d}{dx} x^{n-1}\right)}_{0} + \underbrace{\left(\frac{d}{dx} x\right)}_{0} x^{n-1}$

$= 0 + 0 = 0$. ∎

For $n=1$, $x^n = x^1 \cdot x^0$

cannot apply I.H. to x^1 when $n=1$!

Actual Claim: $\forall n \in \mathbb{N}$, if n is odd, then $2^n \equiv 2 \bmod 3$
& if n is even, then $2^n \equiv 1 \bmod 3$.

Actual Proof: By induction

Base Cases $n=0$. n is even, and
$2^0 = 1 \equiv 1 \bmod 3$.

$n=1$ n is odd, and
$2^1 = 2 \equiv 2 \bmod 3$

IH: For $0 \le k < n$, k odd $\to 2^k \equiv 2 \bmod 3$
k even $\to 2^k \equiv 1 \bmod 3$

Suppose n is odd.
Then $n-2$ is odd, so by I.H.,
$2^{n-2} \equiv 2 \bmod 3$, so

$$2^n = 4 \cdot 2^{n-2} \equiv 4 \cdot 2 \bmod 3$$
$$\equiv 8 \bmod 3$$
$$\equiv 2 \bmod 3.$$

Suppose n is even.
Then $n-2$ is even, so by I.H.
$2^{n-2} \equiv 1 \bmod 3$, so

$$2^n = 4 \cdot 2^{n-2} \equiv 4 \cdot 1 \bmod 3$$
$$\equiv 4 \bmod 3$$
$$\equiv 1 \bmod 3.$$ □