1. Edges of the Hypercube

Recall that $Q_n$ stands for the $n$-dimensional hypercube. Let $\alpha_n$ denote the number of edges in $Q_n$. Answer the following questions.

(a) Choose the correct recurrence relation defining $\alpha_n$. [5 points]

- A. $\alpha_0 = 1$ and $\alpha_n = 2\alpha_{n-1} + 2^n$ for $n \geq 1$ [2 points]
- B. $\alpha_1 = 1$ and $\alpha_n = 2\alpha_{n-1}$ for $n \geq 2$ [1 point]
- C. $\alpha_1 = 0$ and $\alpha_n = 2\alpha_{n-1} + 2^n$ for $n \geq 2$ [1 point]
- D. $\alpha_0 = 0$ and $\alpha_n = 2\alpha_{n-1} + 2^{n-1}$ for $n \geq 1$ [1 point]

(b) Draw three levels of a recurrence tree for $\alpha_n$, with $\alpha_{n-2}$ at the leaves. [5 points]

(c) The closed-form formula for $\alpha_n$ is $n \cdot 2^{n-1}$. [5 points]

2. How many nodes are there in a full-binary tree of depth $d$, if every internal node of the tree has at least one leaf node as a child? [5 points]

- A. $2^{d+1} - 1$
- B. $2^d + 1$
- C. $2d + 1$ [2 points]
- D. $d + 1$
- E. There is not enough information to uniquely determine an answer.
CS 173 (B), Spring 2015, Examlet 4, Part B

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□ D. $\alpha_1 = 0$ and $\alpha_n = 2\alpha_{n-1} + 2^n$ for $n \geq 2$ \[1 \text{ points}\]

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