1. Regular Graphs. [6 points]
   Recall that we say that a graph is \(d\)-regular if every node in the graph has degree exactly \(d\). If \(G\) is 5-regular, then \(G\) has at least 6 nodes and 15 edges (and there is a 5-regular graph with that many nodes and edges).

2. Chromatic Number. [8 points]
   Consider the graphs \(G_1\) and \(G_2\) represented by the following adjacency matrices.
   \[
   G_1 = \begin{bmatrix}
   0 & 0 & 1 & 1 & 1 \\
   0 & 0 & 1 & 1 & 1 \\
   1 & 1 & 0 & 1 & 1 \\
   1 & 1 & 1 & 0 & 0 \\
   1 & 1 & 1 & 0 & 0 
   \end{bmatrix}
   \quad \quad \quad 
   G_2 = \begin{bmatrix}
   0 & 0 & 1 & 1 & 1 \\
   0 & 0 & 1 & 1 & 1 \\
   1 & 1 & 0 & 0 & 0 \\
   1 & 1 & 0 & 0 & 0 \\
   1 & 1 & 0 & 0 & 0 
   \end{bmatrix}
   \]
   Then, \(\chi(G_1) = 3\) and \(\chi(G_2) = 2\).

3. Induction. [6 points]
   Suppose that the following claims have been proven regarding some predicate \(P\) defined over all integers.
   - \(P(0)\) is true, \(P(1)\) is false and \(P(2)\) is false.
   - For all integers \(k\), \(P(k)\) is true if and only if \(P(k + 3)\) is true.

   Then what is the most that we can say about \(P\)? (Select one.)
   
   □ A. \(\forall n \in \mathbb{N}, n \equiv 0 \pmod{3} \rightarrow P(n)\)
   □ B. \(\forall n \in \mathbb{Z}, n \equiv 0 \pmod{3} \rightarrow P(n)\)
   □ C. \(\forall n \in \mathbb{N}, n \equiv 0 \pmod{3} \leftrightarrow P(n)\)
   □ D. \(\forall n \in \mathbb{Z}, n \equiv 0 \pmod{3} \leftrightarrow P(n)\)
   □ E. None of the above is necessarily true.
1. **Regular Graphs.**

Recall that we say that a graph is $d$-regular if every node in the graph has degree exactly $d$. If $G$ is 5-regular, then $G$ has at least 6 nodes and 15 edges (and there is a 5-regular graph with that many nodes and edges).

2. **Chromatic Number.**

Consider the graphs $G_1$ and $G_2$ represented by the following adjacency matrices.

$$
G_1 = \begin{bmatrix}
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 0
\end{bmatrix}
\quad
G_2 = \begin{bmatrix}
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 0
\end{bmatrix}
$$

Then, $\chi(G_1) = 2$ and $\chi(G_2) = 4$.

3. **Induction.**

Suppose that the following claims have been proven regarding some predicate $P$ defined over all integers.

- $P(1)$ is true, $P(2)$ is false and $P(3)$ is false.
- For all integers $k$, $P(k)$ is true if and only if $P(k + 3)$ is true.

Then what is the most that we can say about $P$? (Select one.)

- A. $\forall n \in \mathbb{Z}^+, n \equiv 1 \pmod{3} \rightarrow P(n)$
- B. $\forall n \in \mathbb{Z}, n \equiv 1 \pmod{3} \rightarrow P(n)$
- C. $\forall n \in \mathbb{Z}^+, n \equiv 1 \pmod{3} \leftrightarrow P(n)$
- D. $\forall n \in \mathbb{Z}, n \equiv 1 \pmod{3} \leftrightarrow P(n)$
- E. None of the above is necessarily true.