1. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) and \( g : \mathbb{R} \rightarrow \mathbb{R} \) be defined as follows:

\[
\begin{align*}
  f(x) &= \begin{cases} 
    x + 1 & \text{if } x < -1 \\
    x - 1 & \text{if } x > 1 \\
    0 & \text{otherwise.}
  \end{cases} \\
  g(x) &= \begin{cases} 
    x - 1 & \text{if } x < 0 \\
    x + 1 & \text{if } x > 0 \\
    0 & \text{otherwise.}
  \end{cases}
\end{align*}
\]

Mark all the correct choices below: \([8 \text{ points}]\)

- A. \( f \) is one-to-one. \(\square\)
- B. \( g \) is one-to-one. \(\checkmark\)
- C. \( f \) is onto. \(\checkmark\)
- D. \( g \) is onto. \(\square\)

\(\checkmark\) Treat as 4 True/False problems worth 2 point each.

2. For \( f \) and \( g \) as defined above, define \( g \circ f \) and \( f \circ g \). \([8 \text{ points}]\)

**Solution:**

\[
\begin{align*}
  g \circ f(x) &= \begin{cases} 
    x & \text{if } x < -1 \text{ or } x > 1 \\
    0 & \text{otherwise.}
  \end{cases} \\
  f \circ g(x) &= x \text{ for all } x \in \mathbb{R}.
\end{align*}
\]

(Note that \( f \) is an inverse of \( g \). On the other hand, \( f \) is not one-to-one and hence cannot have an inverse.)

\(\checkmark\) 4 points for each part. Full points for describing the functions alternatively (e.g., \( f \circ g \) is the identity function, or just \( f \circ g(x) = x \) without quantifying \( \forall x \in \mathbb{R} \)).

\(\checkmark\) -1 if \( g \circ f \) is given as identity.

\(\checkmark\) Up to 2 points for each part, involving some elements of the correct answer.

3. Given relations \( \sqsubseteq_1 \) and \( \sqsubseteq_2 \) over a set \( S \), define a new relation \( \sqsubseteq_{12} \) over \( S \) as follows:

\( \forall a, b \in S, a \sqsubseteq_{12} b \text{ if (and only if) } a \sqsubseteq_1 b \text{ or } a \sqsubseteq_2 b. \)

Mark all the correct choices below: \([4 \text{ points}]\)
A. If ⊏₁ is reflexive, so is ⊏₁₂.

B. If ⊏₁ is irreflexive, so is ⊏₁₂. (Both need to be irreflexive for ⊏₁₂ to be so.)

C. If both ⊏₁ and ⊏₂ are symmetric, so is ⊏₁₂.

D. If both ⊏₁ and ⊏₂ are anti-symmetric, so is ⊏₁₂. (Consider having \( a \sqsubseteq₁ b \) and \( b \sqsubseteq₂ a \).)

♦ Treat as 4 True/False problems worth 1 point each.
1. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 
  x - 1 & \text{if } x < 0 \\
  x + 1 & \text{if } x > 0 \\
  0 & \text{otherwise.}
\end{cases}$$

$$g(x) = \begin{cases} 
  x + 1 & \text{if } x < -1 \\
  x - 1 & \text{if } x > 1 \\
  0 & \text{otherwise.}
\end{cases}$$

Mark all the correct choices below: [8 points]

- ✓ A. $f$ is one-to-one.
- □ B. $g$ is one-to-one.
- □ C. $f$ is onto.
- ✓ D. $g$ is onto.

♠ Treat as 4 True/False problems worth 2 point each.

2. For $f$ and $g$ as defined above, define $g \circ f$ and $f \circ g$. [8 points]

$$g \circ f(x) = x \text{ for all } x \in \mathbb{R}.$$  
$$f \circ g(x) = \begin{cases} 
  x & \text{if } x < -1 \text{ or } x > 1 \\
  0 & \text{otherwise.}
\end{cases}$$

(Note that $g$ is an inverse of $f$. On the other hand, $g$ is not one-to-one and hence cannot have an inverse.)

♠ 4 points for each part. Full points for describing the functions alternatively (e.g., $g \circ f$ is the identity function, or just $g \circ f(x) = x$ without quantifying $\forall x \in \mathbb{R}$).

♠ -1 if $f \circ g$ is given as identity.

♠ Up to 2 points for each part, involving some elements of the correct answer.

3. Given relations $\sqsubseteq_1$ and $\sqsubseteq_2$ over a set $S$, define a new relation $\sqsubseteq_{12}$ over $S$ as follows:

$\forall a, b \in S, a \sqsubseteq_{12} b$ if (and only if) $a \sqsubseteq_1 b$ or $a \sqsubseteq_2 b$.

Mark all the correct choices below: [4 points]
A. If both $\sqsubset_1$ and $\sqsubset_2$ are symmetric, so is $\sqsubset_{12}$.

B. If both $\sqsubset_1$ and $\sqsubset_2$ are anti-symmetric, so is $\sqsubset_{12}$. (Consider having $a \sqsubset_1 b$ and $b \sqsubset_2 a$.)

C. If $\sqsubset_1$ is reflexive, so is $\sqsubset_{12}$.

D. If $\sqsubset_1$ is irreflexive, so is $\sqsubset_{12}$. (Both need to be irreflexive for $\sqsubset_{12}$ to be so.)

♦ Treat as 4 True/False problems worth 1 point each.