CS 173: Discrete Structures, Spring 2012
Midterm 1 Grading Notes

General Instructions

Below are the rubrics and grading notes for the last two problems on the midterm. An updated document will hopefully contain rubrics and notes for the other problems, unless they are straightforward. If you believe that your exam was graded incorrectly, please do the following:

1. If it is a simple matter of your points not being summed correctly, just bring that to the attention of your section leader, or to any staff member during their office hours. Otherwise, follow steps 2 through 4.

2. Read the official solutions that are available on the Exams web page.

3. Read the rubric listed here, and try to understand why you got the points you did, and why you didn’t get the points you would have liked. Most questions can probably be answered yourself if you understand the correct solution, and how the exams were graded.

4. If you still have questions, go to your section leader or any staff member’s office hours. It will help if you have your questions or concerns written down on a separate piece of paper, clipped or stapled to your exam, and ready to hand in if the concern cannot be dealt with at that moment, or without being addressed by the person who graded the problem. (Please don’t approach the instructors about grading issues before or after lecture, unless you would simply like to submit your written concerns for evaluation later.)

Grading Rubrics
Problem 5

There were five main areas for points in this problem, for a total of 9 points. As there are two versions of this question, the rubric below will be written from the perspective of the Bush version, with Obama version details given in parentheses.

1. Introduction of Variables and Assumptions (IVA): 1 point

Full credit was given for proofs that properly described both the variables and the assumptions needed to prove antisymmetry. Specifically, two elements $a, b$ from $P(M)$ needed to be chosen, and it needed to be stated that $aTb$ and $bTa$ ($aPb$ and $bPa$). Zero points were given for IVA if the conclusion was assumed, or a similar inappropriate assumption was made. This also affected scoring for the Algebra section.

Zero points were given for IVA if a broken definition of antisymmetry was used.

Half a point were given for IVA if the variables were described without the assumptions, or vice versa.

2. Using the definition of the set $P(M)$, as appropriate (Def): 1 point

Full credit was given for proofs that explained what it meant for $a, b$ to be from $P(M)$. That is, it was necessary to express that $a = (x, 2x+1), b = (y, 2y+1)$ ($a = (3y+1, y), b = (3x+1, x)$) where $x, y \in \mathbb{R}^+$. It was acceptable to express such in combination with the IVA step, as many people did.

Some students never applied the definition of $P(M)$, and thus never checked for its validity in their algebra, thereby losing both this point and points from Algebra.

3. Application of the relation $T(P)$, as appropriate (Rel): 2 points

Full credit was given for proofs that explained what it meant for $aTb$ and $bTa$ ($aPb$ and $bPa$). That is, it was necessary to express that $x(2y + 1) \geq y(2x + 1)$ and $y(2x + 1) \geq x(2y + 1)$. ($(3y+1)x \geq (3x+1)y$ and $(3x+1)y \geq (3y+1)x$)

It was acceptable to express this first as $aq \geq pb$, $pb \geq aq$, and later apply the definition of $P(M)$, as many students did.

4. Correct algebra steps (Alg): 4 points

Full credit was given for proofs that progressed from the application of the relation and definition of the set to the necessary conclusion of $a = b$.

A maximum of 2 points were given for proofs that had unclear steps or unproven assertions. This was left to the grader to determine. This also included proofs that followed from the wrong definition of antisymmetry, as we told you explicitly that you should be using the second definition (the contrapositive) to be proving antisymmetry.

A maximum of 2 points were given for proofs that at no point used the definition of $P(M)$. It was necessary to validate not only that the relation was consistent, but that the included set definition was also consistent.
Zero points were given for algebra that arrived at $0 = 0$ or similar, as this is a so-called “U-shaped proof”, and we have explicitly instructed students not to do this.

Zero points were given for approaches involving cases, unless one of those cases eventually asserted $aq = pb$ (or its equivalent using $P (M)$).

Zero points were given for algebra that proceeded from an assumed conclusion.

5. Conclusion (Conc): 1 point

Full credit was given for proofs that asserted $x = y, 2x + 1 = 2y + 1 \rightarrow a = b \ (3y + 1 = 3x + 1, y = x \rightarrow a = b)$, which thus established antisymmetry by definition.

Half a point was given to proofs that fell short of the $a = b$ step; that is, proofs that asserted only $x = y, 2x + 1 = 2y + 1 \ (3y + 1 = 3x + 1, y = x)$ and stopped there.

Zero points were given to conclusions that were not supported by the work leading to them. That is, if a proof merely asserted that the relation was antisymmetric without showing conclusive that it was so, no points were given.

Other notes

- Zero credit was given for proofs relying on examples. These are not proofs.

- A deduction of four points was given for proofs that intentionally set out to prove by contradiction. The directions clearly stated, “You must work directly from the definitions”.

- A deduction of two points was given for proofs that arrived at a contradiction without seeming to have meant to do so. We gave students the benefit of the doubt on this one.

- A deduction of three points was given for proofs that stated that $a \leq b, b \leq a$ constituted a contradiction.

- A deduction of two points was given for proofs wherein the two points were not given as distinct. Some students wanted to reuse variables when applying the definition of the set, e.g. $a = (2x + 1, x), b = (2x + 1, x)$.

- Zero credit was given for proofs relying on calculus details, e.g. statements of increasing functions. The directions clearly stated “using basic rules of algebra”.

- Up to one point may have been deducted for general style - needing more words, legibility, clarity, etc. This was at the discretion of the grader.
Problem 6

The solution to this problem involved five main steps:

1. (3 points) Using the definition of congruence, and divides, to re-express $a$ as a sum of multiples of $r$ and $m$. Note that the instructions specifically required that you use the definition of divides. The three points were allocated as follows:

   (a) 1 point for noting that by the definition of congruence, $r|a-m$, or $m|a-r$, depending on which version of the exam you had.

   (b) 1 point for using the definition of divides to say that there exists an integer $k$ such that $rk = a-m$ or $mk = a-r$, depending on which version you had. (If the part of the instructions that told you to use the definitions of divides was circled, it means that you failed to do this.)

   (c) 1 point for concluding that $a = rk + m$ or $a = r + mk$, depending on which version you had. It was also possible to get a point for concluding something else was true that was used later. If you made it to this point, you usually will have received 2 points, no matter how you got here. Most people received 2 or 3 points for this work.

2. (2 points) Picking an arbitrary element of $L(a,m)$, and expressing it in terms of the definition of $L(a,m)$. This was broken down by giving 1 point for saying something like “Let $p$ be an arbitrary element of $L(a,m)$”, and 1 additional point for saying “Then $p = ax + my$ for some integers $x$ and $y$”. It was possible to get these two points in a single sentence “Let $p = ax + my$ for some integers $x$ and $y$ be an element of $L(a,m)$”. Or even “Let $ax + my$ be an element of $L(a,m)$ for some integers $x$ and $y.” If you didn’t choose some element of $L(a,m)$, then you did not receive any points. (If the part of the instructions telling you to choose an element of the smaller set was circled, it means you failed to do this.)

3. (2 points) Re-expressing it by substituting the result obtained in the first step. This involved noting that $ax + my$ can be rewritten $(rk + m)x + my$ or $(r + mk)x + my$, depending on which version you had. One point was given for the substitution, and one more for regrouping to the form $r(kx) + m(x + y)$ or $rx + m(kx + y)$.

4. (1 point) For noting that the coefficients of $r$ and $m$ in the previous part were integers, and thus concluding that the element is in $L(r,m)$.

5. (1 point) Available for overall clarity and style.

Not all proofs followed the above sketch. Nonetheless, it was possible to get points for including some of the steps above. For example, most students received 2 points for expressing $a$ as a linear combination of $r$ and $m$ somehow, as in part (a).

Many attempts ignored the specific instructions to choose an element of the smaller set ($L(a,m)$) and show it was in the larger set ($L(r,m)$). Instead, they attempted to re-express the definition
of \(L(a,m)\) as follows (shown for one version of the exam; the other is comparable):

\[
L(a,m) = \{as + mt : s,t \in \mathbb{Z}\} \\
= \{(rk + m)s + mt : s,t \in \mathbb{Z}\} \\
= \{r(ks) + m(s + t) : s,t \in \mathbb{Z}\} \\
= \{rx + my : x,y \in \mathbb{Z}\} = L(r,m)
\]

There were several problems with this. First, the questions specifically asked you to choose an element of \(L(a,m)\), not to re-express the definition. Second, the last step is incorrect, since not every possible \(x\) can be used - only those that are of the form \(ks\) (i.e., those divisible by \(k\)), where \(k\) was the integer such that \(rk = a - m\).

Nonetheless, many of these attempts did manage to get about 4 or 5 points for demonstrating knowledge in parts (a) and (c) above.

More problematic though were attempts that began as above, but then somehow confused sets with numbers, dropping the set braces, and writing something like this:

\[
L(a,m) = as + mt \\
= (rk + m)s + mt \\
L(r,m) = rs + mt
\]

and then attempts were made to prove that \((rk + m)s + mt \subseteq rs + mt\), which makes little sense, because these are numbers, and \(\subseteq\) is a set operation. If you see “#” and/or “set” written on your exam, it is flagging a problem like this - where you are comparing one with the other, or using a set operation on a number, of a numeric operation on a set.

Some papers did similar manipulations, and some resulted with expressions of the form \(a \subseteq r\), which “was true because \(a \leq r\)”. Needless to say, there were many problems with such solutions, and few points were given.

Finally, several attempts did not fit any approach discussed above, and points were given based on the grader’s ability to find evidence of knowledge being applied towards the main goal. It was unlikely that such attempts would receive more than a few points, e.g., for demonstrating understanding of congruence and divides, as in part (a) above.