Math jargon

Margaret M. Fleck

Fall 2010

Mathematicians write in a dialect of English that differs from everyday English and from formal scientific English. To read and write mathematics fluently, you need to be aware of the differences.

1 Strange technical terms

Many technical terms, abbreviations, and shorthand symbols are easy to track down (e.g. in the lecture notes). Perhaps they belong to an obvious topic area. Perhaps they are used so heavily that no one forgets what they mean. Perhaps they are easily found on Wikipedia. But others seem to appear mysteriously out of nowhere.

abuse of notation The author is using a notation that doesn't technically comply with the rules of mathematical writing, but is very convenient or conventional. For example, using "if" in a a definition where we really mean "if and only if." Or writing $f(2, 3)$ for the result of applying f to the pair $(2, 3)$. Strictly we should have written $f((2, 3))$, but no one except a computer program would ever do that.

$\exists!$ This is the unique existence quantifier. $\exists! x, P(x)$ means that there is exactly one x for which $P(x)$ holds.

By symmetry A variant wording for "without loss of generality."

IH Inductive hypothesis, as in a proof by induction.

QED Short for "quod erat demonstrandum." This is just the Latin translation of "which is what we needed to show." The Latin and the English versions are both polite ways to tell your reader that the proof is finished.

tuple A generalization of "pair," "triple," etc to larger numbers of items. A k-tuple is an ordered sequence of k items. One occasionally hears "2-tuple" in place of "pair."

WLOG, WOLOG, without loss of generality We're stipulating an extra fact or relationship, but claiming that it doesn't actually add any new information to the problem. For example, if we have two real variables x and y, with identical properties so far, it's ok to stipulate that $x \leq y$ because we're simply fixing which of the two names refers to the smaller number.

NTS Need to show. As in "we need to show that all horses have four legs."

2 Odd uses of normal words

In addition to the obvious technical terms (e.g. "rational number"), some words are used rather differently in mathematics than in everyday English.

Clearly See "obviously."

Consider The author is about to pull an example or constant or the like out of thin air. Typically, this part of the proof was constructed backwards, and the reasons for this step will not become apparent until later in the proof.

Obviously Perhaps it really is obvious. Or perhaps the author didn't feel like working out the details (or forgot them in the heat of lecture). Or, infamously, perhaps he just wants to intimidate the audience.

Or The connective "or" and its variations (e.g. "either...or") leave open the possibility that both statements are true. If a mathematician means to exclude the possibility that both are true ("exclusive or"), they say so explicitly. In everyday English, you need to use context to determine whether an "or" was meant to be inclusive or exclusive.

Has the form E.g. "if x is rational, then x has the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$. Means that the definition of this type of object forces the object to have this specific internal structure.

Plainly See "obviously."

Proper For example, "proper subset" or "proper subgraph." Used to exclude the possibility of equality. Very similar to "strict."

Recall The author is about to tell you something basic that you probably ought to know, but he realizes that some of you have gaps in your background or have forgotten things. He's trying to avoid offending some of the audience by suggesting they don't know something so basic, while not embarrassing everyone else by making them confess that they don't.

Similarly You can easily fill in these details by adapting a previous part of the proof, and you'll just get bored if I spell them out. Occasionally misused in a manner similar to "obviously."

Strict A "strict" relationship is one that excludes the possibility of equality. So "strictly less than" means that the two items can't be equal. A "strictly increasing" function never stays on the same value. Compare "proper."

Suppose not A stereotypical way to start a proof by contradiction. It's shorthand for "Suppose that the claim is not true."

Unique There is only one item with the specified properties. "There is a unique real number whose square is zero."

Vacuously true The claim is true, but only because it's impossible to satisfy the hypothesis. Recall that in math, an if/then statement is considered true if its hypothesis is true. Vacuously true statements often occur when definitions are applied to examples that are very small (e.g. the empty set) or lacking some important feature (e.g. a graph with no edges, a function whose domain is the empty set).

Well-defined A mathematical object is well-defined if its definition isn't buggy. The term usually appears in a context where the definition might look ok at first glance but could have a subtle bug.

3 Constructions

Mathematicians also use certain syntactic constructions in ways that aren't quite the same as in normal English.

If/then statements An if/then statement with a false hypothesis is considered true in mathematics. In normal English, such statements are rare and it's not clear if they are true, false, or perhaps neither.

Imperatives A mathematical construction is often written as a recipe that the author and reader are carrying out together. E.g. "We can find a real number x such that $x^2 < 47$. The picture behind this is that the author is helping you do the mathematics, not doing it by himself while you watch. Verbs in the imperative form are sometimes actions that the reader should do by himself (e.g. "prove XX" on a problem set) but sometimes commands that the reader should do by following explicit instructions from the author. E.g. "define $f : \mathbb{N} \rightarrow \mathbb{N}$ by $f(n) = 2n$" tells you to define a function, but you are being told exactly how to do it. On a problem set, this would be background information, not a question you need to answer.

Two variables If a mathematician sets up two variables with different names, e.g. x and y, he leaves open the possibility that they might be equal. In normal English, giving two things different names suggests very strongly that they are different objects.

No resetting Do not change the value of a variable in the middle of a proof. Instead, use a fresh variable name. For example, suppose you know that $p = 3jk + 1$ in a proof about divisibility. To simplify the form of the equation, it's tempting to reset j to be three times its original value, so that you have $p = jk + 1$. This isn't done in standard mathematical writing. Instead, use a fresh variable name, e.g. $s = 3j$ and then $p = sk + 1$.

Variable names can be re-used when the context changes (e.g. you've started a new proof). When working through a series of examples, e.g. a bunch of example functions to be discussed, rotate variable names, so that the new definition appears after the reader has had time to forget the old definition.

Variable names Variable names are single-letter, e.g. f but not foo. The letter can be adorned with a wide variety of accents, subscripts, and superscripts, however.

4 Unexpectly normal

Mathematicians are, of course, underlyingly speakers of normal English. So, sometimes, they apply the normal rules of conversational English even when these aren't part of the formal mathematical system.

names of variables In principle, any letter can be used as the name for any variable you need in a proof, with or without accent marks of various sorts. However, there are strong conventions favoring certain names. E.g. x is a real number, n is an integer, f is a function, and T is a set. Observe what names authors use in your specific topic area and don't stray too far from their conventions.

topics of equations Proofs are conversations about particular objects (e.g. polynomials, summations). Therefore, equations and other statements are often about some specific topic object, whose properties are being described. When this is the case, the topic belongs on the lefthand side, just like the subject of a normal sentence.