

## CS 173: Discrete Mathematical Structures, Spring 2009 Honors Homework 1

Due by 4pm on Wednesday 18 February. Please give to Margaret or Eric or push it under the door of Margaret's office (3214 Siebel).

A binary operation on a set  $A$  is a function that takes two elements of  $A$  and returns an element of  $A$ . For example, addition and multiplication are binary operations on the set of real numbers.

Suppose that  $*$  is a binary operation on a set  $A$ . Then  $*$  is *commutative* if

$$\forall x, y \in A, x * y = y * x$$

and  $*$  is *associative* if

$$\forall x, y, z \in A, x * (y * z) = (x * y) * z$$

For example, multiplication of real numbers is commutative and associative, but subtraction is neither. However, the two properties are actually independent of one another.

- Let's define the operation  $\odot$  on the real numbers as follows:

$$x \odot y = 2(x + y)$$

Is  $\odot$  commutative? associative? Prove that your answers are correct.

- A  $2 \times 2$  matrix is a 2-by-2 table of real numbers, e.g.

$$\begin{bmatrix} 47.0 & -32.3 \\ -1.7 & 104.2 \end{bmatrix}$$

The formula for multiplying two  $2 \times 2$  matrices is as follows:

$$\begin{bmatrix} a & b \\ c & c \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Is  $2 \times 2$  matrix multiplication commutative? associative? Prove that your answers are correct. (Prove them using only high-school algebra and stuff we've seen in this course. Don't invoke facts about matrices that you might have seen in previous courses.)

3. We've found that computer science majors are often shy about talking to faculty members. So here is a little exercise designed to help you meet some of them.

Please find a computer science faculty member in person. A list of people who have agreed to talk to students is posted on the honors web page (and probably have people added to it over the next couple days).

Use the departmental web pages to find out their office location and good times to drop by and catch them for 2 minutes (e.g. office hours, right after their class lets out).

Ask them two questions. First, what is their area of research? (Just the main label for their subarea of CS, e.g. "Computer Graphics," not a long description.) Second, what is their favorite flavor of ice cream? Write the results on a piece of paper, and have them sign the paper.