

CS 173, Spring 2009

Homework 5

Due **At Elaine Wilson's office** by 11am on Friday, March 13th, 2009
(Total point value: 50 points.)

There will be no lecture on the day this is due, because of Engineering Open House. So you will be dropping off your homeworks at Elaine Wilson's office, 3229 Siebel. Notice that the homework is due at 11am. Although there's a bit of fuzz in that deadline, Elaine leaves for lunch at 12, at which point I'll take the folders away.

1. Recursive definition [10 points]

- (a) Consider the function h defined by the following recursive definition. Compute $h(x)$ for x from 0 to 10.

$$h(0) = 0$$

$$h(1) = 1$$

$$h(2) = 1$$

$$h(n) = h(n-1) + h(n-2) - h(n-3) \text{ if } n \geq 3$$

- (b) Consider the function g defined by the following recursive definition. Compute $g(x)$ for x from 1 to 10.

$$g(1) = 1$$

$$g(2) = 2$$

$$g(n) = g(n-1) + g(n-2) \text{ if } n \geq 3$$

2. Induction [10 points]

Use induction to prove that the following formula holds, for any positive integer n .

$$\sum_{k=1}^n k2^k = (n-1)2^{n+1} + 2$$

3. Recursive definition proof [10 points]

Let's define a function f as follows

$$f(0) = 1$$

$$f(n) = f(n-1) + 4n$$

Use induction to prove $f(n) = 2n^2 + 2n + 1$.

4. Strong induction [10 points]

The Noble Kingdom of Frobboz has two coins: 3-cent and 7-cent.¹ Use strong induction to prove that the Frobboznics can make any amount of change ≥ 12 cents using these two coins.

You must use strong induction.

¹They used to have a 1-cent coin, but inflation has made it essentially useless.

5. More fun with function composition [10 points]

Suppose that A, B, C are sets and suppose there are functions $f : B \rightarrow C$ and $g : A \rightarrow B$.
Claim: If $f \circ g$ is surjective and f is injective, then g is surjective.

- (a) Why did we require f to be injective? Give a concrete counter-example which shows why this condition is necessary.
- (b) Prove the claim.

You may find it useful to draw a diagram (like on pages 139 and 141 of the textbook) of the three sets and the three functions, to help develop an intuitive picture of what's happening before you try to do the proof.