CS 173, Spring 2009
Homework 3

Due *in class* on Friday, February 20th, 2009
(Total point value: 50 points.)

1. **Euclidean algorithm [4 points]**

   Trace the execution of the Euclidean algorithm (lecture 9, p 229 in Rosen) on the inputs 1224 and 850. That is, give a table showing the values of the main variables (x, y, r) for each pass through the loop.

2. **Numbers...[10 points]**

   (a) Convert $(10110111)_2$ to decimal notation.

   (b) Convert $(1111101011011101101)_2$ to hexadecimal notation.

   (c) Convert $(CAFE8)_{16}$ to binary notation.

   (d) Convert $(1234)_{10}$ to binary notation.

   (e) How many bits are required to represent a positive integer n in binary notation? Express your answer as a function of n.

   (f) How many hexadecimal "digits" are required to represent the positive integer n? Express your answer as a function of n.

   (g) Suppose you have 127 coins and 10 bags. How can you divide the coins among the bags so that you can give out any number from 1 to 127 coins without opening the bags?

3. **Proof by contradiction [10 points]**

   Use proof by contradiction to prove the following claim. Do this directly from the definition of the "divides" relation (section 3.4 of Rosen, lecture 7), i.e. do not use any other facts about divides that may have been proved in class or in the text.

   > For all integers x and y, if $3x + 5y = 47$ then at least one of x and y is not divisible by 7.

4. **Set operations [12 points]**

Let's define sets as follows:

$$A = \{68, 28\}$$
$$B = \{\text{rain}, \text{snow}, \text{sun}\}$$
$$C = \{\text{water}, \text{ice}\}$$
$$D = \{\{\text{water}\}, \{\text{milk}\}\}$$
$$E = \{(\text{water}, \text{ice})\}$$
$$F = \{\text{ink}\}$$

For each of the following expressions, list the elements of the set or calculate the value (as appropriate).

(a) $\mathbb{P}(B)$

(b) $\mathbb{P}(E)$

(c) $(A \times F) \cup D$

(d) $\mathbb{P}(C) - D$

(e) $\mathbb{P}(C) \cap \mathbb{P}(E)$

(f) $|\mathbb{P}(A \cup B) \cup \mathbb{P}(D \cup E)|$

Recall that $\mathbb{P}(X)$ is the power set of X. Show your work.

5. **A Semi-Numerical Algorithm [10 points]**

The following algorithm takes as input a natural number c and a list of $n+1$ natural numbers a_0,...,a_n and yields a single natural number p as output.

procedure DoSomething(c, a_0, ..., a_n: natural numbers)
$p := a_n$
for $i := 1$ to n
    $p := (p \times c) + a_{n-i}$

(a) If $c = 10$ and the input list is $1, 2, 3, 4$ what is the output p?

(b) Describe in words what DoSomething does. That is, give descriptive names to the input c and the inputs $a_0, \ldots, a_n$, and then explain how the output value is related to these inputs.

(c) How many multiplications and additions does the algorithm perform given a list of $n+1$ natural numbers a_0,...,a_n as input? Include only the operations plainly visible on the last line of the code. E.g. do not include the additions required to increment the loop index i in your answer. Express your answer as a function of n.

(d) Consider the binary expansion $b_n 2^n + b_{n-1} 2^{n-1} + ... + b_1 2^1 + b_0$. If we evaluate this expansion by performing the addition, multiplication, and exponentiation operations exactly as written, how many multiplications and additions are performed? In your answer, count each exponentiation operation as a set of multiplications (e.g. evaluating 2^3 requires 2 multiplications). Express your answer as a function of n.

6. **Fun with paradoxes [4 points]**

In a children's book (whose identity I'm deliberately hiding), our hero is blocked by a powerful man who says

> We will play a game to decide which way you will die. You may say one thing, and one thing only. If what you say is true, I will strangle you with my bare hands. If what you say is false, I will cut off your head.

Conveniently, the powerful man is like many such characters in fantasy and science fiction: he self-destructs if he makes promises that turn out to contain a logical contradiction. How should our hero answer the question, so as to create a contradiction?