CS 173, Spring 2009
Homework 2

Due *in class* on Friday, February 13th, 2009
(Total point value: 50 points.)

1. **[8 points]** Primes

   (a) Express the numbers 350, 105, and 64 as products of primes.

   (b) Compute GCD$(350, 105)$, GCD$(105, 64)$, and LCM$(350, 64)$. Feel free to give large results as products of primes; multiplying them out is not necessary.

   (c) According to the definitions given in the book (or in lecture 7), which integers are neither prime nor composite?

2. **[7 points]** Divisibility, congruence mod k

   Which of the following statements are correct? Show work or give brief explanations for your answers.

   (a) $-6 \mid 30$

   (b) $30 \mid -6$

   (c) $6 \mid -30$

   (d) $-19 \equiv 7 \pmod{13}$

   (e) $-6 \equiv 6 \pmod{4}$

   (f) $-6 \equiv 6 \mod 24$

   (g) $0 \equiv 17 \mod 17$

3. **[10 points]** Direct proof using congruence mod k

   In the book, you will find several equivalent ways to define congruence mod k. For this problem, use the following definition: for any integers x and y and any positive integer m, $x \equiv y \pmod{m}$ if there is an integer k such that $x = y + km$.

   Using this definition prove that, for all integers x, y, p, q and m, with $m > 0$, if $x \equiv p \pmod{m}$ and $y \equiv q \pmod{m}$, then $(x^2 + y^2) \equiv (p^2 + q^2) \pmod{m}$.

4. **[10 points]** Proof by contradiction

   Consider the following claim

   $\forall x \in \mathbb{R}$, if $x^2 - 3x + 2 > 0$, then $x > 2$ or $x < 1$.

(a) State the negation of this claim, moving all instances of “not” onto individual propositions and then making them disappear by inverting the inequalities.

(b) Prove the claim using proof by contradiction.

5. **[15 points]** Another direct proof

For any two real numbers x and y, the harmonic mean of x and y is $H(x, y) = \frac{2xy}{x+y}$. This is a form of averaging that penalizes the case when either of the inputs is very small, often used for combining two performance numbers when evaluating a computer program.

(a) This definition has a small but important bug. What is it?

(b) The more familiar arithmetic mean is $M(x, y) = \frac{x+y}{2}$. When is $H(x, y)$ equal to $M(x, y)$?

(c) Rephrase your answer to (b) in the form $\forall x, y \in \mathbb{R}, P(x, y) \rightarrow Q(x, y)$, for some suitable choice of predicates $P(x, y)$ and $Q(x, y)$. (Hint: in this case, the predicates are equations.)

(d) Prove that your answer to (b) is correct.