CS 173, Spring 2008
Homework 1

Due *in class* on Friday, February 6th, 2009
(Total point value: 50 points.)

1. **[9 points]** Translate the following sentences into propositional logic, making the meaning of your propositional variables clear. See page 11 of the textbook for some examples of translating English sentences into propositional logic.

   (a) Neither the storm blast nor the flood did any damage to the house.

   (b) If global warming isn't controlled, more forests will die in the Pacific Northwest.

   (c) Drivers should neither drive over 65 miles per hour nor cross the red light, or they will get a ticket.

2. **[11 points]**

   Show that the following logical equivalences are correct. For (a), use a truth table. For (b) and (c), use the logical equivalences given on pages 24 and 25 of the textbook (in Tables 6 through 8).

   Note that for (b) and (c) you should be very picky about explicitly using associative laws, commutative laws, double negation laws, etc. Refer to the examples on page 26 of Rosen to get an idea of what your proofs should look like.

   (a) (3 points) $(p \oplus q) \equiv ((\neg p) \rightarrow q) \wedge (q \rightarrow (\neg p))$

   (b) $((\neg p) \rightarrow (r \vee q)) \equiv ((\neg r) \rightarrow ((\neg p) \rightarrow q))$

   (c) $(p \rightarrow (r \rightarrow q)) \equiv (p \rightarrow q) \vee \neg r$

3. **[9 points]** Rewrite the following statements using predicate logic shorthand, making clear the meaning of your predicates and simple propositions, as well the types of any variables bound by quantifiers. For example, the English sentence

   Anyone who thinks 173 is fun is either crazy or an instructor .

can be rewritten as follows, assuming that P is the set of all people.

$$\forall x \in P,\ \big[\textit{ThinksFun}(x, 173) \rightarrow (\textit{Crazy}(x) \vee \textit{Instuctor}(x))\big]$$

For this problem, assume that any use of the word "or" refers to inclusive or ($\vee$), not exclusive or ($\oplus$).

(a) No prime number except 3 is divisible by 3.

(b) Every CS 173 student has to solve a homework and at least one CS 173 student must shovel snow.

(c) It is easy to drive when it does not rain or snow.

4. **[9 points]**

Rewrite each of the following propositions as unambiguous English sentences. The relevant predicates are defined as follows: Again, let's let P be a set of all people.

- $A(x)$ means "x is teaching CS 173."
- $T(x)$ means "x is taking CS 473."
- $F(x)$ means "x has a Facebook page."
- $C(x)$ means "x likes to cook."

For example, the proposition $\exists x \in P\, [A(x) \wedge C(x)]$ could be translated as "There is a person who is teaching CS 173 and likes to cook."

(a) $\forall x \in P,\ [T(x) \rightarrow F(x)]$

(b) $\exists z \in P,\ [T(z) \wedge A(z)]$

(c) $\neg\forall x \in P,\ [T(x) \rightarrow (F(x) \vee C(x))]$

5. **[12 points]**

Give the negation of the following logical expressions, using logical equivalences to move the "not" operators onto the smallest elements possible. For example, to negate $\forall x[P(x) \rightarrow Q(x)]$, we first negate the whole thing $\neg\forall x[P(x) \rightarrow Q(x)]$, then convert this to $\exists x[\neg(P(x) \rightarrow Q(x))]$, and finally to $\exists x[P(x) \wedge \neg Q(x)]$. (For simplicity, we've omitted the domains for the quantified variables.)

(a) $\forall x[P(x) \vee Q(x)]$

(b) $\exists y[P(y) \wedge (Q(y) \wedge R(y))]$

(c) $\exists x[(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x)]$

(d) $\forall z[P(z) \to (\neg Q(z) \to P(z))]$