CS 173: Discrete Mathematical Structures, Spring 2008
Homework 0

Due *in class* on Friday, January 30, 2009

Submit your solutions for this homework *in class* on Friday, January 30. Please make sure to read the course policies on homework *before* writing up your homework.

Grading: The questions on this homework have a total value of 50 points with a 5 point bonus question.

---

1. **Logarithms, exponents, complex numbers [6 points]**
Simplify the following expressions as much as possible, **without using a calculator (either hardware or software)**. Do not approximate. Express all rational numbers as fractions. For complex numbers use i to represent the value $\sqrt{-1}$.

   (a) $\sqrt[3]{\sqrt{\pi^{1200}}}$

   (b) $\frac{(2^9 \times 2^7)^4}{256}$

   (c) $\log_5 625^n$

   (d) $(\log_3 63) - (\log_3 7)$

   (e) $\frac{\log_{10} 4096}{\log_{10} 2}$

   (f) $(1-i)(2-i)(3-i)$

2. **Numbers of all kinds...[4 points]**
In mathematical writing it is customary to denote certain common sets of numbers using special symbols (often a single letter). For example, the set of rational numbers is typically denoted as $\mathbf{Q}$. We will follow the textbook conventions, listed on the inside cover of your textbook, for assigning symbols to these sets.

   (a) What letter is used to denote the set of numbers $\{..., -2, -1, 0, 1, 2, ...\}$? What is this set called?

   (b) How is the set of numbers $\{1, 2, ...\}$ denoted and what is this set called?

   (c) (2 points) What is the formula for computing $\frac{a+bi}{b+ci}$ where $i = \sqrt{-1}$?
   Hint: look it up on the web or in a reference book.

3. **Floors and ceilings [5 points]**

   (a) What is $\lceil \lfloor \frac{1}{2} \rfloor + \lfloor \frac{1}{3} \rfloor - \frac{1}{3} + \frac{1}{2} \rceil$?

   (b) What is $\lfloor -63.5 \rfloor$?

   (c) What is $\lfloor 5.5 \rfloor - \lceil -5.5 \rceil$

   (d) Is $\lceil 2x \rceil \leq \lceil x \rceil + \lceil x - \frac{1}{2} \rceil$ for all real numbers x? Just answer yes or no, you don't need to justify your answer.

   (e) Is $\lceil x - y \rceil = \lceil x \rceil - \lceil y \rceil$ for all real numbers x and y? Justify your answer.

4. **Functions [5 points]**
   Suppose $F(x) = x^2 - 4x$ and $G(x) = x + 4$ and $H(x) = x^2 - 4$.

   (a) What is $F(G(z))$?

   (b) What is $F(G(G(G(G(G(-20))))))$?

   (c) Express $F(x) + H(G(y))$ as a single function.

   (d) Simplify $\frac{G(F(x))}{H(x)}$ as much as possible.

5. **Sums and products [10 points]** Please show your work in deriving the solutions to the following questions. Note that in these questions, i is an index variable and is not representing an imaginary number.

   (a) Express $S(n) = \sum_{i=0}^{n-1} (2i + 1)$ as a simple function of the variable n.

   (b) Express $P(n) = \prod_{i=1}^{n} \frac{3(i+1)}{3i}$ as a simple function of the variable n.

   (c) Rewrite $\sum_{i=1}^{n} \frac{2^i}{i^2}$ as a sum which has an index from 0 to $n - 1$.

   (d) Express $S(n) = \sum_{i=0}^{n} (4i + 1)$ as a simple function of the variable n.

   (e) Express $S(n) = \sum_{i=1}^{n} \frac{1}{i^2(i^2+1)}$ as a simple function of the variable n.
   hint: can you rewrite $\frac{1}{i^2(i^2+1)}$ as a difference of two fractions?

6. **Algorithms [10 points]**
The word *algorithm*, derived from the name of the ninth-century Persian mathematician al-Khowārizmī, refers to a step-by-step method for solving a problem. We will use *pseudo-code* to describe algorithms in this class. This should save you time, since you don't have to learn another full programming language. It also will make the algorithm descriptions more concise and understandable. Our version of pseudo-code resembles a very minimal imperative programming language and is described in Appendix 3 of the textbook. When reading an algorithm description, if anything about the pseudocode convention is unclear you should consult the textbook.

The following algorithm takes as input an arbitrary list of n real numbers a_1,...,a_n and yields a single real number p as output.

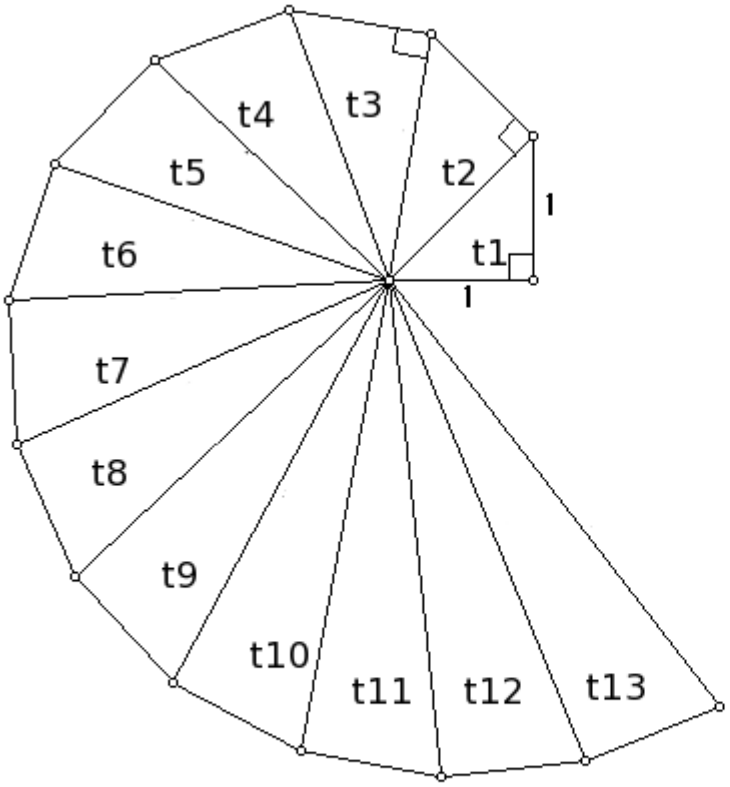
procedure DoSomething(a_1,..., a_n)
$p := a_1$
for $i := 2$ to n
    $temp := p$
    for $j := 1$ to a_i
        $p := p \times temp$

(a) If the initial list is $10, 2, 2, 2$ what is the output p?

(b) How many multiplications does the algorithm perform given a list of n real numbers a_1,...,a_n as input?

(c) If the input is a list of 3 real numbers a_1,a_2,a_3 express the output of the algorithm as mathematical function of the list elements.

7. **Spirals [10 points]**
Consider the spiral shown in the figure below. This construction starts with a right-angled triangle which has sides of length 1. Successive right-angled triangles are added on with a base of length 1 and a side with length equal to the hypotenuse of the previous triangle.



(a) If we denote the first triangle as t_1, how long is the hypotenuse of the nth triangle t_n?

(b) Write a formula for the total area of the first n triangles using summation notation.

(c) ***5 point bonus question:*** For $n > 1$, do you think the total area of the first n triangles can be exactly represented by a digital computer? Briefly and clearly explain why or why not in five or fewer sentences.