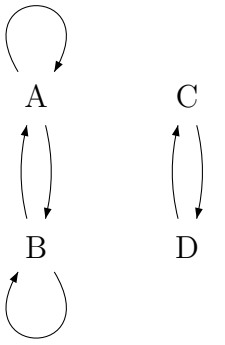
CS 173: Discrete Structures, Spring 2009
Quiz 3 Solutions

1. (10 points)

   Check all boxes which correctly characterize each relation, leaving the other boxes blank. (If you change your answer, make it very clear when you've meant to uncheck a box.)

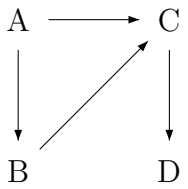


Reflexive: ☐ **Irreflexive**: ☐

Symmetric: ☑ **Antisymmetric**: ☐

Transitive: ☐

   It's not transitive because C is related to D which is related to C, but C is not related directly to itself. (The same with D as well.)



Reflexive: ☐ **Irreflexive**: ☑

Symmetric: ☐ **Antisymmetric**: ☑

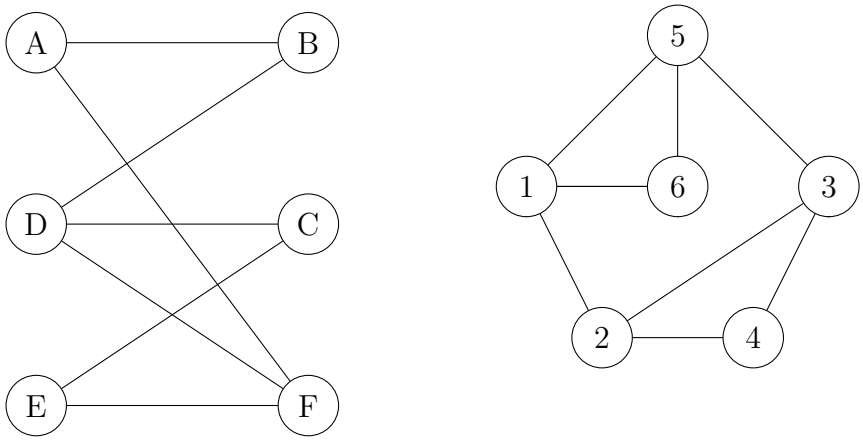
Transitive: ☐

2. (3 points) Suppose a graph has 100 vertices which all have degree 4. How many edges are there in the graph?

   Solution: The sum of the vertex degrees is 400. So, by the handshaking theorem, the graph must contain 200 edges.

3. (4 points)

Are the two graphs shown here isomorphic? If so, write down a mapping of the vertices of one graph to the vertices of the other that demonstrates the isomorphism. If not, give a reason why they are not isomorphic.



Solution: They aren't isomorphic. The lefthand graph contains four vertices with degree 2 (A, B, C, and E) and two vertices of degree 3, whereas the righthand graph contains only two vertices of degree 2 (4 and 6) and four vertices of degree 3. Also the lefthand graph has 7 edges but the righthand graph has 8 edges.

4. (3 points)

In the polynomial $(2x - 3y)^{20}$, what is the coefficient of the term x^5y^{15}? (Please do not attempt to simplify your formula.)

Solution: By the binomial theorem, this coefficient is $2^5(-3)^{15}\begin{pmatrix} 20 \\ 15 \end{pmatrix}$.

Or $2^5(-3)^{15}\begin{pmatrix} 20 \\ 5 \end{pmatrix}$. (The two expressions are equal.)

5. (5 points)

Imagine a game in which a fair coin is flipped until either heads comes up or N flips have occurred. In general, if the ith flip comes up heads you win $3i$ dollars. So, if you flipped $TTTH$ you would get 12 dollars, if you flipped TTH you would get 9 dollars, but getting H on the first flip would only win you 3 dollars. However, if N flips occur without heads ever appearing, you win nothing.

What are your expected winnings? The probability of heads on a given flip is 0.5 and the sample space consists of the possible sequences of flips that can happen given the rules of the game. Show your work and briefly explain your answer.

You should express your answer as a summation. We don't think there is a simple way to convert the summation to a closed form solution.

Solution: The probability of winning on the i'th flip is the probability of getting $i-1$ tails followed by a head. This is $(\frac{1}{2})^i$.

The expected value is the sum of the probability of each outcome times the payoff for that outcome. We can ignore the outcome consisting of N tails because the payoff is zero in that case. So the expected value is

$$\sum_{1}^{N} 3i \left(\frac{1}{2}\right)^i$$

It is possible to reduce this to a closed form, but not in any reasonable amount of time using the techniques we've seen in this class.