CS 173: Discrete Structures, Fall 2008
Quiz 2 (Wednesday 22 October)

NAME:

NETID:

This quiz has 3 pages containing 5 questions. You have 20 minutes to finish. Showing your work may increase partial credit in case of mistakes.

1. (1 point) Give the day and time when your assigned discussion section meets. (If you've recently switched sections, give either one.)

2. (5 points) Mark each of the statements as "true" or "false".

   (a) n^2 is $\Omega(n \log n)$

   (b) $87 \log n$ is $O(n^3)$

   (c) $n!$ is $O(100n^3 + 35)$

   (d) $2^k + 2^{10}$ is $O(2^k)$

   (e) The rational numbers have the same cardinality as the integers.

3. (4 points) Negate the following statement, moving the "not" onto individual (non-complex) propositions.

   For every $p \in \mathbb{Z}$, there is a $q \in \mathbb{Z}$, if $p > q$, then $q^3 + p!$ is a multiple of 7.

4. (9 points) Each of the following definitions or claims has a significant flaw. Explain briefly what's wrong with each:

   (a) If $f : \mathbb{N} \to \mathbb{N}$ is an increasing function, then f is one-to-one.

   (b) If $f : A \to B$ and $g : B \to C$ are functions, then $f \circ g : A \to C$ is defined by $(f \circ g)(x) = f(g(x))$.

   (c) Suppose that f and g are functions whose domain and range are the real numbers. Then f is $O(g)$ if there is a constant real number m such that $|f(x)| \leq m\,|g(x)|$ for every $x \geq k$.

5. (6 points) Use mathematical induction to prove that

$$\sum_{k=1}^{n} \frac{1}{2^k} = 1 - \frac{1}{2^n}$$

for any integer $n \geq 1$. Notice that the summation starts at 1. So the first term in the summation is $\frac{1}{2}$. You must do this problem by induction, not using some other proof technique.