# CS 173: Discrete Structures, Spring 2009
# Quiz 1 Solutions

1. (1 point) Give the day and time when your assigned discussion section meets. State explicitly if you have switched sections very recently.

   **Solution:** The right answer varies from person to person. But section names like "AD1" aren't an answer to the question as stated.

2. (4 points) Compute the following quantities.

   (a) $\lfloor -3.7 \rfloor =$ **Solution:** -4

   (b) $-7 \bmod 3 =$ **Solution:** 2

3. (4 points) Give a closed-form expression for the following summation.

$$\sum_{k=2}^{n+1} k =$$

   **Solution:** $= (\sum_{k=1}^{n+1} k) - 1 = \frac{(n+1)(n+2)}{2} - 1$

4. (7 points) Are the following equivalences, formulas, and claims correct? Write "yes" next to the ones that work for all input values. Write "no" next to the ones that fail in some cases.

   (a) $0 \mid 14$ **Solution:** No. For any integer k, 0 times k is 0, not 14.

   (b) $5 \mid -15$ **Solution:** Yes. $5 \times -3 = -15$

   (c) $n^{m^n} = n^{mn}$ **Solution:** No. See the section on manipulating exponentials. Notice that you have to be careful with how you group expressions. $n^{m^n}$ is read as $n^{(m^n)}$. But if we force the opposite grouping using parentheses, we get $(n^m)^n$, which is indeed equal to $n^{mn}$.

   (d) $\neg(p \rightarrow q) \equiv p \rightarrow \neg q$ **Solution:** No. The lefthand side is equivalent to $p \wedge \neg q$ and the righthand side is equivalent to $\neg p \vee \neg q$. So if $p$ is false, the lefthand side is false but the righthand side is true.

(e) $-3 \equiv 4 \pmod 7$ **Solution:** Yes. $4-(-3)=7$ which is a multiple of 7.

(f) Zero is neither even nor odd. **Solution:** No. Zero is even, because it's a multiple of 2.

(g) The statement "$\forall x \in \mathbb{N}, x < 0 \rightarrow x^2 < 0$" is false. **Solution:** No. This statement is vacuously true, because $x < 0$ is false for all natural numbers $x$.

5. (3 points) Complete the following definition, using precise mathematical English and/or notation.

   An integer $p$ is odd if and only if

   **Solution:** there is an integer $k$ such that $p = 2k + 1$.

6. (3 points) Negate the following statement, rephrasing so that each "not" is on an individual (non-complex) proposition.

   For all integers $x$ and $y$, $x < y$ implies that both $x^2 < y^2$ and $x - y < 0$.

   **Solution:** Not (For all integers $x$ and $y$, $x < y$ implies that both $x^2 < y^2$ and $x - y < 0$)

   which is equivalent to: There are integers $x$ and $y$, such that not ($x < y$ implies that both $x^2 < y^2$ and $x - y < 0$)

   which is equivalent to: There are integers $x$ and $y$, such that $x < y$ and not ($x^2 < y^2$ and $x - y < 0$)

   which is equivalent to: There are integers $x$ and $y$, such that $x < y$ and (not $x^2 < y^2$ or not $x - y < 0$)

   which is equivalent to: There are integers $x$ and $y$, such that $x < y$ and ($x^2 \geq y^2$ or $x - y \geq 0$)

7. (3 points) State the Fundamental Theorem of Arithmetic.

   **Solution:** Every integer greater than 1 can be written as a product of primes, in a way that's unique (given that we don't care about the order in which we write the primes).