CS 173, Spring 2008
Midterm 2, 5 November 2008

INSTRUCTIONS (read carefully)

- Fill in your name, netid, and discussion section time below. Also write your netid on the other pages (in case they get separated).

NAME:

NETID:	DISC:

- There are 5 problems, on pages numbered 1 through 5. Make sure you have a complete exam.
- The point value of each problem is indicated next to the problem, and in the table below.
- Points may be deducted for solutions which are correct but excessively complicated, hard to understand, or poorly explained.
- Brief explanations and/or showing work may increase partial credit for buggy answers.
- You have 50 minutes to finish the exam.
- Turn in your exam at the front. Show your ID to the proctors.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam. No notes or electronic devices of any kind are allowed. Do all work in the space provided, using the backs of sheets if necessary. See the proctor if you need more paper.
- Please bring any apparent bugs to the attention of the proctors.
- After the midterm is over, discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem	1	2	3	4	5	total
Possible	12	9	10	9	10	50
Score						

Problem 1: True/false (12 points)

Label each of the following statements as true or false.

(a) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x < y$

(b) $n + \log_2 n$ is $O(n)$

(c) $n!$ is $O(2^n)$

(d) $n \log_8 n$ is $\Theta(n \log_2 n)$

(e) The function $g : \mathbb{Z} \to \mathbb{R}$ defined by $g(x) = x + 1$ is onto.

(f) The real numbers are not countable.

Problem 2: Short answer (9 points)

(a) The number of ways to pick a k-element subset from a set containing n elements is written $C(n,k)$ or $\binom{n}{k}$. Give an equation for computing this quantity using factorials.

(b) Let $f : \mathbb{N} \to \mathbb{Z}$ be defined by $f(x) = 7x + 12$. Prove that f is one-to-one.

Problem 3: Recursive definition (10 points)

(a) Here is a recursive definition of a set S, which contains pairs of numbers:

1) $(2, 1) \in S$ and $(1, 1) \in S$
2) If $(x, y) \in S$, then $(xy, 1) \in S$
3) If $(x, y) \in S$ and $(p, q) \in S$, then $(x, p) \in S$.

Give a non-recursive definition for the set S. Explain briefly and/or show your work.

(b) Find a closed-form solution for the following recurrence relation with the given initial condition. A closed-form solution is a function $T(n)$ that yields that same values as the recurrence relation but is non-recursive. You should be able to find the solution by *unrolling* the recurrence and then applying a formula you have seen before to find a closed form for a summation. Show your work.

$T(n) = T(n-1) + 2n$ with initial condition $T(0) = 0$

Problem 4: Algorithms (9 points)

The following algorithm is a recursive form of binary search. It takes as input an arbitrary list of n real numbers $a_1,\ldots,a_n$ and determines if a given number x is in the list. If the number is in the list, the function returns the position of x, if x is not in the list it returns the value 0. In the following code i and j are integers indicating the current search range (i.e. $i = 6$ and $j = 10$ means a_6 through a_{10} are to be searched).

(a) Fill in the 2 blank lines below with pseudo-code that will correctly execute binary search recursively.

procedure binary search(x,i,j,a_1,...,a_n)
$m := \lfloor (i + j)/2 \rfloor$
if $(x = a_m)$ then
    location $:= m$
else if $(x > a_m$ and $i < m)$ then

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else if $(x < a_m$ and $m < j)$ then

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else *location* $:= 0$

(b) Suppose the function $T(n)$ returns the number of operations needed by binary search to find x (in the worst case) in a list of length n. Write a recurrence relation (recursive formula) for $T(n)$. Your recurrence should count the comparison and arithmetic operations done by binary search, but it need not be exact: you can use letters such as c in your formula to represent constant numbers.

Problem 5: Induction (10 points)

Let's define a sequence of numbers x_n as follows:

Base: $x_1 = 1$, $x_2 = 7$

Induction: for every $n \geq 2$, $x_{n+1} = 7x_n - 12x_{n-1}$

Use induction to prove that $x_i = 4^n - 3^n$ for every integer $n \geq 1$. Hint: the algebra in the inductive step should work out easily.