CS 173, Spring 2008
Midterm 1, 1 October 2008

INSTRUCTIONS (read carefully)

- Fill in your name, netid, and discussion section time below. Also write your netid on the other pages (in case they get separated).

NAME:

NETID:	**DISC:**

- There are 6 problems, on pages numbered 1 through 6. Make sure you have a complete exam.
- The point value of each problem is indicated next to the problem, and in the table below.
- Points may be deducted for solutions which are correct but excessively complicated, hard to understand, or poorly explained.
- You have 50 minutes to finish the exam.
- Turn in your exam at the front. Show your ID to the proctors.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam. No notes or electronic devices of any kind are allowed. Do all work in the space provided, using the backs of sheets if necessary. See the proctor if you need more paper.
- Please bring any apparent bugs to the attention of the proctors.
- After the midterm is over, discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem	1	2	3	4	5	6	total
Possible	8	9	9	8	8	8	50
Score							

Problem 1: Short Answer (8 points)

(a) Suppose p, q, and r are propositional variables (i.e. they can be true or false). Using propositional logic, write a proposition that is true when p and q are true and r is false, but is false otherwise. You may use any or all of the logical operators $\vee, \wedge, \neg, \rightarrow$. Show that your proposition gives the desired results by writing out a truth table for it.

(b) Use the Euclidean Algorithm to compute the GCD(111,201). Show every step the algorithm takes to compute the answer.

Problem 2: Set theory (9 points)

List all the elements of the following sets. Recall that $\Re$ is the set of real numbers and that $\mathbb{P}(S)$ denotes the power set of a set S. Show your work.

(a) $\{x : x \epsilon \Re \wedge (3.5 \leq x < 10) \wedge (\lfloor x \rfloor = \lceil x \rceil)\}$

(b) $\mathbb{P}(\{8\}) \times \{4, 5\}$

(c) $\mathbb{P}(\{27\}) - \{27, 38\}$

Problem 3: Set theory (9 points)

Suppose A is a non-empty set containing n elements and B is a non-empty set containing m elements. Remember that $\mathbb{P}(S)$ denotes the power set of a set S. Show your work.

(a) What is $|\mathbb{P}(A \times B)|$?

(b) What is $|\mathbb{P}(A)||\mathbb{P}(B)|$?

(c) Under what conditions is $|\mathbb{P}(A \times B)| = |\mathbb{P}(A)||\mathbb{P}(B)|$?

Problem 4: Remembering definitions (8 points)

(a) Suppose that A and B are sets and f is a function from A to B. State precisely what it means for f to be one-to-one.

(b) Suppose that p and q are positive integers. State precisely what it means for p and q to be relatively prime.

Problem 5: Logic and proof mechanics (8 points)

(a) Give the negation of the following statement. Write your answer out in words and move the negatives (e.g. "not") so they are on individual predicates (e.g. "has two wheels").

> For any vehicle V, if V has two wheels and V's wheel diameter is more than 16 inches, then V is a bicycle.

(b) What's wrong with the following proof that $\overline{A \cup B} = \overline{A} \cap \overline{B}$? (Hint: you are looking for a major flaw, not a small issue with wording or justifications.)

> Let A and B be sets. Let x be an element of $\overline{A \cup B}$. By the definition of set complement, it is not the case that x is in $A \cup B$. By the definition of set union, this means that the following is not the case: x is in A or x is in B. By de Morgan's law for logic, this is equivalent to saying that x is not in A and x is not in B. So (by the definition of set complement), x is in $\overline{A}$ and x is in $\overline{B}$. So x is in in $\overline{A} \cap \overline{B}$ by the definition of set intersection.

Problem 6: Writing a proof (8 points)

Recall the following definition: Given any positive integer m, the integers a and b are *congruent modulo* m if and only if there is an integer k such that $a = b + km$. a is congruent to b modulo m is written as $a \equiv b \pmod{m}$.

Prove that, for any integers a, b, c, and d and any positive integer m,

$$\text{if } a \equiv c \pmod{m} \text{ and } b \equiv d \pmod{m}, \text{ then } a + b \equiv c + d \pmod{m}.$$

Prove this directly using the above definition, together with basic logic and algebra. Do not use other facts about modular arithmetic proved in class or in the book.