

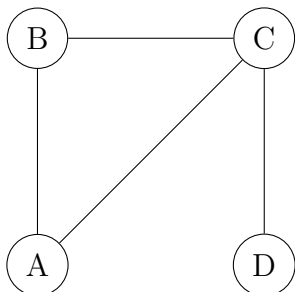
# Week 12 Additional Tutorial Problems

## Partitions

a) Let  $A = \{2, 5, 7, 8, 13, 21\}$ , and define  $p : A \rightarrow \mathbb{P}(A)$  by  $p(n) = \{s \in A \mid \gcd(s, n) \neq 1\}$ .  
Let  $M = \{p(s) \mid s \in A\}$ .

- What are the elements of  $M$ ?
- Is  $M$  a partition of  $A$ ? Explain why or why not.

b) Consider this graph on vertex set  $X = \{A, B, C, D\}$ :



Define  $D : \mathbb{N} \rightarrow \mathbb{P}(X)$  by  $D(n) = \{v \in X \mid \text{degree of node } v \text{ is } n\}$ . Let  $S = \{D(n) \mid n \in \mathbb{N}\} = \{D(0), D(1), D(2), \dots\}$ .

- What are the elements of  $S$ ?
- Is  $S$  a partition of  $X$ ? Explain why or why not.

## Set-valued functions

Define  $f : \mathbb{P}(Z) \rightarrow \mathbb{P}(Z)$  by  $f(S) = \{n/2 \mid n \in S \text{ and } n \text{ is even}\}$ .

- a) Is  $f$  one-to-one?
- b) Is  $f$  onto?

## Counting

Let  $n, k$  be integers with  $n \leq k$ . Compute the number of positive integer solutions to the equation  $\sum_{i=1}^n x_i = k$ . *Hint: this is similar to problem 17.5a.*