Homework on Graphs and Trees
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Problem 1. Among connected simple graphs whose sum of vertex degrees is 20:

a) What is the smallest possible number of vertices?

b) What is the largest possible number of vertices?\(^1\)

Briefly justify each answer.

Problem 2. Prove that in a simple graph, if there are any closed walks with odd length, any shortest of them is a cycle.\(^2\)

Problem 3. Suppose every vertex in a graph has degree at least \(k\). Explain why the graph has a path of length \(k\).

Problem 4. Prove that every \(n\)-vertex graph other than \(K_n\) has chromatic number less than \(n\). \(^3\)

Problem 5. Solve Problem 12.24 in the textbook. (You do not need to turn in the drawing of the graph, but you should still draw it.)

\(^1\) Hint: it may help to think in terms of "connected components" - the biggest possible subsets of the graph that are still connected. For example, a connected graph has 1 connected component (the whole graph), and a graph with \(n\) vertices and no edges has \(n\) connected components (each vertex is its own).

\(^2\) Hint: Prove by contradiction. A closed walk which is not a cycle can be written as \(\overline{f \overline{g} \overline{b} \overline{h}}\) for some walks \(f, g, h\) and repeated vertex \(b\).

\(^3\) Hint: find an \((n - 1)\)-coloring.