Propositional Logic:

Valid: All rows of truth table evaluate to T

Satisfiable: Some row of truth table evaluates to T

Predicate Logic:

Valid: The formula evaluates to T in all domains of discourse

Satisfiable: The formula evaluates to T in some domain of discourse

$$\boxed{\underbrace{\forall x}_{\exists x} \quad P(x) \text{ OR } \text{NOT}(P(x))} \qquad \text{VALID}$$

$$\underbrace{\forall x. \quad x^2 \geq 0} \qquad \text{SATISFIABLE}$$

$$\forall x \quad \underbrace{x \geq 0}$$

$$x = \sqrt{-1} \qquad x^2 = -1 \not\geq 0$$

$$\boxed{\exists x. \quad x^2 \geq 0} \quad \checkmark \text{ Complex numbers}$$

Universe: $\{0, 1\}$, $\quad 0 \geq 1$

$$a \cdot b = 1$$

$$\forall x \in \mathbb{R} \quad x^2 \geq 0 \qquad \boxed{\forall x \quad x^2 \geq 0}$$

$$\forall x \cdot x = x - 1$$

Lecture 5: More Proofs

Date: September 6, 2019.

Definition 1. An integer n is **even** if there is an integer k such that $n = 2k$. An integer n is **odd** if there is an integer k such that $n = 2k + 1$.

Problem 1. Prove: If n is an integer such that $3n + 2$ is odd then n is odd.

$n = 2(3g+2)$

Consider an arbitrary integer n s.t $3n+2$ is odd

There some integer k s.t $3n+2 = 2k+1$

$$n = \frac{2k-1}{3}$$

$P \Rightarrow Q$. Contrapositive: $\neg Q \Rightarrow \neg P$

If n is an integer n is even then $3n+2$ is even.

Consider an arbitrary integer n s.t n is even

There is some integer k s.t. $n = 2k$.

$$3n+2 = 3(2k)+2 = 2\underbrace{(3k+1)}_{\text{integer}}$$

$\Rightarrow$ $3n+2$ is even

Problem 2. Prove: An integer n is odd if and only if n^2 is odd.

P IFF Q $\equiv$ $(P \Rightarrow Q)$ AND $(Q \Rightarrow P)$

IF n is odd then n^2 is odd

If n^2 is odd then n is odd