Lecture 2: Propositional Logic

Date: August 28, 2019.

Definition 1. A **proposition** is a statement that is either true or false.

Examples: 5 is prime

Non examples: When is this going to end? Go there!

A **propositional variable/Boolean variable** is a variable that takes value either T (true) or F (false).

Building Complex Propositions from Propositions

NOT (RANDS)

P	NOT(P)
F	T
T	F

P	Q	P AND Q
F	F	F
F	T	F
T	F	F
T	T	T

P	Q	P OR Q
F	F	F
F	T	T
T	F	T
T	T	T

LOGIC DESIGN

T → 1

F → 0

NOT → $\bar{\ }$

AND → ·

OR → +

PROG

NOT → !

AND → &&

OR → ||

LOGIC

T → T

F → $\perp$

NOT → $\neg$

AND → $\wedge$

OR → $\vee$

IMPLIES → $\Rightarrow$

P	Q	P IMPLIES Q
F	F	T
F	T	T
T	F	F
T	T	T

if pigs could fly then your bank account is secure.

If Goldbach conjecture is true then for any real number x, $x^2 \geq 0$

CONTRAPOSITIVE: NOT (Q) IMPLIES NOT (P)

CONVERSE: Q IMPLIES P

P	Q	P IFF Q
F	F	T
F	T	F
T	F	F
T	T	T

For any real number x

$x^2 \leq 4$ iff $-2 \leq x \leq 2$ ✓

(P IMPLIES Q)	AND	(Q → P)
T	T	T
T	F	F
F	F	T
T	T	T

Logical Equivalence

Problem 1. Show that the following logical expressions are the same: (a) P IMPLIES Q and (NOT(P)) OR Q (b) P IMPLIES Q and (NOT(Q)) IMPLIES (NOT(P)), (c) NOT(P OR Q) and (NOT(P)) AND (NOT(Q)).

P	Q	P IMPLIES Q	NOT (P)	OR Q
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	T	F	T

P IMPLIES Q $\boxed{\text{NOT (Q) IMPLIES NOT(P)}}$

NOT(NOT(P)) $\equiv P$
De Morgan NOT(P OR Q) $\equiv$ (NOT(P)) AND (NOT(Q))
NOT(P AND Q) $\equiv$ (NOT(P)) OR (NOT(Q))
NOT(P IMPLIES Q) $\equiv$ P AND (NOT(Q))

P AND (Q AND R) $\equiv$ (P AND Q) AND R
P OR (Q OR R) $\equiv$ (P OR Q) OR R
P OR (Q AND R) $\equiv$ (P OR Q) AND (P OR R)
P AND (Q OR R) $\equiv$ (P AND Q) OR (P AND R)

Question 1. Are the following pairs equivalent?

- P OR Q and Q OR P YES
- P AND Q and Q AND P YES
- P IMPLIES Q and Q IMPLIES P NOT
- P IMPLIES (Q IMPLIES R) and (P IMPLIES Q) IMPLIES R