
LECTURE 1: INTRODUCTION TO PROOFS

Date: August 26, 2019.

Question: What is a proof?

Way establish truth

A mathematic proof of a proposition is a series of logical deductions starting from a set axioms

"5 is a prime", "Champaign is the capital of Illinois"

"What is the capital of Illinois?"

Problem 1. Is $n^2 + n + 41$ prime, for any n that is a non-negative integer?

$\{0, 1, 2, \dots\} = \mathbb{N}$

n	$n^2 + n + 41$	
0	$0 + 0 + 41 = 41$	✓
1	$1 + 1 + 41 = 43$	✓
2	$4 + 2 + 41 = 47$	✓
20	$400 + 20 + 41 = 461$	✓
41	$41^2 + 41 + 41$	✗

Problem 2 (Euler's Conjecture). The equation

$$a^4 + b^4 + c^4 = d^4$$

$$a^2 + b^2 = c^2$$

has no solution when a, b, c, d are positive (non-zero) integers.

Noam Elkies proved this was false

$$a = 95800, \quad b = 217519, \quad c = 414560, \quad d = 422481$$

Proposition 1 (Fermat's Last Theorem). There are no positive integers x, y, z such that

$$x^n + y^n = z^n$$

for some integer $n > 2$.

Theorem 2 (Four Color Theorem). Every map can be colored with 4 colors, so that adjacent regions have different colors.

Guthrie in 1852

1976 — Haken and Appel

Conjecture: (Goldbach) Every even integer > 2 is the sum of two primes.

Problem 3. Does the following program halt for all positive integer, initial values for n ?

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while ( $n \neq 1$ )
  if  $n$  is even
     $n \leftarrow n/2$ 
  else
     $n \leftarrow 3n + 1$ 
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