Lecture 7: Sets

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Set: An unordered collection of objects.

$\emptyset = \{\}$	$\mathbb{N}$
$A = \{0, 2, 4, 6\}$	$\mathbb{Z}$
$B = \{\mathsf{B}, \mathsf{C}, \mathsf{D}, \mathsf{E}, \mathsf{F}, \mathsf{J}, \mathsf{K}, \mathsf{P}, \mathsf{Q}, \mathsf{R}, \mathsf{S}, \mathsf{T}, \mathsf{V}\}$	$\mathbb{Q}$
$C = \{\{0\}, \{2\}, \{4\}, \{6\}\}$	$\mathbb{R}$
$D = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$	$\mathbb{C}$

Membership: A set is defined by its **members**. $x \in A$ means "x is a member of A".

Question 1. Which of the following are true?

1. (a) $0 \in \emptyset$, (b) $\emptyset \in \emptyset$, (c) $\mathsf{A} \in \emptyset$?
2. (a) $0 \in A$, (b) $\{0\} \in A$, (c) $\emptyset \in A$?
3. (a) $0 \in C$, (b) $\{0\} \in C$, (c) $\{\{0\}\} \in C$?
4. (a) $\emptyset \in D$, (b) $\{\emptyset\} \in D$, (c) $\{\{\emptyset\}\} \in D$?

Containment: $A \subseteq B$ (A is contained in B) iff $\forall x[x \in A \text{ IMPLIES } x \in B]$.

Question 2. Which of the following are true?

$\emptyset \subseteq \emptyset$ $\qquad \emptyset \subseteq \mathbb{N}$

$\mathbb{N} \subseteq \mathbb{N}$

$C \subseteq A$ $\qquad A \subseteq C$

Set Builder Notation: $\{x \in A \mid P(x)\}$ defines the set of elements in A such that $P(x)$ is true.

$$E = \{n \in \mathbb{N} \mid n \text{ is even}\} = \{n \in \mathbb{N} \mid \exists k \in \mathbb{N}(n = 2k)\}$$
$$F = \{x \in \mathbb{R} \mid \exists a, b \in \mathbb{Z}(b \neq 0) \text{ AND } (x = \tfrac{a}{b})\} = \mathbb{Q}$$

Set Operations: Let X and Y be sets.

$X \cup Y = \{x \mid (x \in X) \text{ OR } (x \in Y)\}$

$X \cap Y = \{x \mid (x \in X) \text{ AND } (x \in Y)\}$

$X - Y = \{x \mid (x \in X) \text{ AND } (x \notin Y)\}$

$\overline{X} = U - X$, where U is the "universal set/domain of discourse" (when understood)

Question 3. What is

$A \cup C$ $\qquad A \cap C$ $\qquad A - C$

$C \cap \emptyset$

$C \cup \emptyset$

Cartesian Product: $X \times Y$ consists of all ordered pairs (x, y) where $x \in X$ and $y \in Y$, i.e., $X \times Y = \{(x, y) \mid (x \in X) \text{ AND } (y \in Y)\}$.

Example 1. $\{0, 1, 2\} \times \{a, b, c\} =$
$\{a, b, c\} \times \{0, 1, 2\} =$
$\emptyset \times D =$
$A \times C =$

Power Set: $\text{pow}(X) = \{Y \mid Y \subseteq X\}$

Question 4. $\text{pow}(\{0, 1, 2\}) =$
$\text{pow}(\emptyset)$ is (a) $\emptyset$, (b) $\{\emptyset\}$, (c) $\{\emptyset, \{\emptyset\}\}$, (d) not defined.
$\text{pow}(\{\emptyset\})$ is (a) $\emptyset$, (b) $\{\emptyset\}$, (c) $\{\emptyset, \{\emptyset\}\}$, (d) not defined.

Set Equality: Two sets X and Y are equal if they have the same elements, i.e., for every x, $x \in X$ IFF $x \in Y$, i.e., $X \subseteq Y$ AND $Y \subseteq X$.

Problem 1. Prove that for any sets X, Y, Z,

$$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z).$$

Cardinality (of finite sets): $|X| =$ number of elements in X.

Example 2. $|\emptyset| =$ $\quad$ $|A| =$ $\quad$ $|D| =$ $\quad$ $|\{0, 1, 1, 2, 2\}| =$
$|A \times B| =$