Lecture 24: Summing Series

Date: October 30, 2019.

Arithmetic Progression. A sequence of (real) numbers such that difference between successive elements is the same. It is of the form

$$a, a+d, a+2d, \dots a+nd, \dots$$

a is the **initial term** and d is the **common difference**.

Proposition 1. *Let $a_1, a_2, \dots a_n$ be an arithmetic progression with initial term a and common difference d. Then $a_i = a + (i-1)d$, and*

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} (a + (i-1)d) = \frac{n(a_1 + a_n)}{2} = \frac{n(2a + (n-1)d)}{2}.$$

Corollary 2. $\sum_{i=1}^{n} i = n(n+1)/2$

Geometric Progression. A sequence of (real) numbers such the ratio of successive elements is the same. It is of the form

$$a, ar, ar^2, \dots ar^n, \dots$$

a is the **initial term** and r is the **common ratio**.

Proposition 3. *Let $a_1, a_2, \dots a_n$ be a geometric progression with initial term a and common ratio r. Then $a_i = ar^{i-1}$. If $r = 1$, $\sum_{i=1}^{n} a_i = na$. If $r \neq 1$,*

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r - 1}.$$

Proposition 4. *If* $|r| < 1$ *then*

$$\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}$$

Proposition 5.

$$\sum_{i=1}^{n} ix^i = \frac{x(1-x^n)}{(1-x)^2} - \frac{nx^{n+1}}{1-x}$$