# Lecture 4: Quantifiers and Proofs

Date: September 4, 2019.

**Definition 1.** A **predicate** is a proposition that depends on the value of variables.

**Universal Quantification**
$\forall x \in \mathbb{Z}.\ x^2 \geq 0$

**Existential Quantification**
$\exists x \in \mathbb{Z}.\ x^2 - 4 = 0$

**Definition 2.** An integer $n$ is **even** if there is an integer $k$ such that $n = 2k$. An integer $n$ is **odd** if there is an integer $k$ such that $n = 2k + 1$.

**Problem 1.** Prove: If $n$ is an odd integer then $n^2$ is is odd.

**Definition 3.** An integer $n$ is a **perfect square** if there is an integer $k$ such that $n = k^2$.

**Problem 2.** Disprove: Any integer is the sum of two perfect squares.