
LECTURE 2: PROPOSITIONAL LOGIC

Date: August 28, 2019.

Definition 1. A **proposition** is a statement that is either true or false.

A **propositional variable/Boolean variable** is a variable that takes value either T (true) or F (false).

Building Complex Propositions from Propositions

P	$\text{NOT}(P)$

P	Q	$P \text{ AND } Q$

P	Q	$P \text{ OR } Q$

P	Q	$P \text{ IMPLIES } Q$

P	Q	$P \text{ IFF } Q$

Logical Equivalence

Problem 1. Show that the following logical expressions are the same: (a) $P \text{ IMPLIES } Q$ and $(\text{NOT}(P)) \text{ OR } Q$
(b) $P \text{ IMPLIES } Q$ and $(\text{NOT}(Q)) \text{ IMPLIES } (\text{NOT}(P))$, (c) $\text{NOT}(P \text{ OR } Q)$ and $(\text{NOT}(P)) \text{ AND } (\text{NOT}(Q))$.

$$\text{NOT}(\text{NOT}(P)) \equiv P$$

$$\text{NOT}(P \text{ OR } Q) \equiv (\text{NOT}(P)) \text{ AND } (\text{NOT}(Q))$$

$$\text{NOT}(P \text{ AND } Q) \equiv (\text{NOT}(P)) \text{ OR } (\text{NOT}(Q))$$

$$\text{NOT}(P \text{ IMPLIES } Q) \equiv P \text{ AND } (\text{NOT}(Q))$$

$$P \text{ AND } (Q \text{ AND } R) \equiv (P \text{ AND } Q) \text{ AND } R$$

$$P \text{ OR } (Q \text{ OR } R) \equiv (P \text{ OR } Q) \text{ OR } R$$

$$P \text{ OR } (Q \text{ AND } R) \equiv (P \text{ OR } Q) \text{ AND } (P \text{ OR } R)$$

$$P \text{ AND } (Q \text{ OR } R) \equiv (P \text{ AND } Q) \text{ OR } (P \text{ AND } R)$$

Question 1. Are the following pairs equivalent?

- $P \text{ OR } Q$ and $Q \text{ OR } P$
- $P \text{ AND } Q$ and $Q \text{ AND } P$
- $P \text{ IMPLIES } Q$ and $Q \text{ IMPLIES } P$
- $P \text{ IMPLIES } (Q \text{ IMPLIES } R)$ and $(P \text{ IMPLIES } Q) \text{ IMPLIES } R$