Lecture 1: Introduction to Proofs

Date: August 26, 2019.

Question: What is a proof?

Problem 1. Is $n^2 + n + 41$ prime, for any n that is a non-negative integer?

Problem 2 (Euler's Conjecture)**.** The equation

$$a^4 + b^4 + c^4 = d^4$$

has no solution when a, b, c, d are positive (non-zero) integers.

Proposition 1 (Fermat's Last Theorem)**.** *There are no positive integers x, y, z such that*

$$x^n + y^n = z^n$$

for some integer $n > 2$.

Theorem 2 (Four Color Theorem)**.** *Every map can be colored with 4 colors, so that adjacent regions have different colors.*

Conjecture: (Goldbach) Every even integer > 2 is the sum of two primes.

Problem 3. Does the following program halt for all positive integer, initial values for n?

```
while (n ≠ 1)
    if n is even
        n ← n/2
    else
        n ← 3n + 1
```