LECTURE 32: PROBABILITY

Date: November 20, 2019.

Problem 1. Suppose we roll a (fair) black die and a (fair) white die. What is the probability that they sum to 7 or 11?

Probability Spaces. Consists of

Sample Space, a set S of possible outcomes of an experiment

Probability Distribution, a function $\mathsf{Pr} : S \to [0, 1]$ that assigns a positive real weight proportion or probability to each outcome such that $\sum_{x \in S} \mathsf{Pr}[x] = 1$.

An **event** $E \subseteq S$ is a subset of outcomes. The probability of an event E is $\mathsf{Pr}[E] = \sum_{x \in E} \mathsf{Pr}[x]$.

Problem 2. Suppose a biased coin, whose probability of showing heads is q, is tossed 30 times. What is the probabiity of seeing 15 heads?

A probability space is said to be **uniform** if $\mathsf{Pr}[x] = \mathsf{Pr}[y]$ for all outcomes x, y. Then $\mathsf{Pr}[E] = \frac{|E|}{|S|}$.

Problem 3. In a class containing 95 students, what is the probability that two people share the same birthday? Assume that all possible birthdays are equally likely.

Probability Rules from Set Theory.

- **Sum Rule.** If $E_1, E_2, \ldots E_n$ are pairwise disjoint sets, then

$$\mathsf{Pr}[\bigcup_{i=1}^{n} E_i] = \sum_{i=1}^{n} \mathsf{Pr}[E_i]$$

- **Complement Rule.** $\mathsf{Pr}[\overline{A}] = 1 - \mathsf{Pr}[A]$.
- **Difference Rule.** $\mathsf{Pr}[B - A] = \mathsf{Pr}[B] - \mathsf{Pr}[A \cap B]$.
- **Inclusion-Exclusion Rule.** $\mathsf{Pr}[A \cup B] = \mathsf{Pr}[A] + \mathsf{Pr}[B] - \mathsf{Pr}[A \cap B]$.
- **Boole's Inequality.** $\mathsf{Pr}[A \cup B] \leq \mathsf{Pr}[A] + \mathsf{Pr}[B]$.
- **Monotonicity Rule.** If $A \subseteq B$ then $\mathsf{Pr}[A] \leq \mathsf{Pr}[B]$.