Lecture 28: Permutations and Combinations

Date: November 8, 2019.

Permutations. A **permuation/arrangement** of n objects is an ordering of the objects. The number of permutations of n *distinct* objects is $n \times (n-1) \times \cdots \times 1 = n!$.

Problem 1. A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $\{1, 2, 3, 4, 5\}$ is *heavy-tailed* if $a_1 + a_2 < a_4 + a_5$. How many heavy tailed permutations are there?

Problem 2. How many orderings of the top 3 finishers are there, in a 10 horse race?

Observation. Number of ways of ordering r objects out of a set containing n objects is

$$P(n, r) = n \times (n-1) \times \cdots \times (n-(r-1)) = \frac{n!}{(n-r)!}.$$

k-to-1 Functions. A function $f : A \to B$ is k-to-1 if exactly k elements of the domain are mapped to every element of the codomain, i.e., for every $b \in B$, $|\{a \in A \mid f(a) = b\}| = k$.

Division Rule. If $f : A \to B$ is a k-to-1 function then $|A| = k|B|$.

Problem 3. How many ways are there to place two identical rooks on a chessboard so that they do not share a row or column?

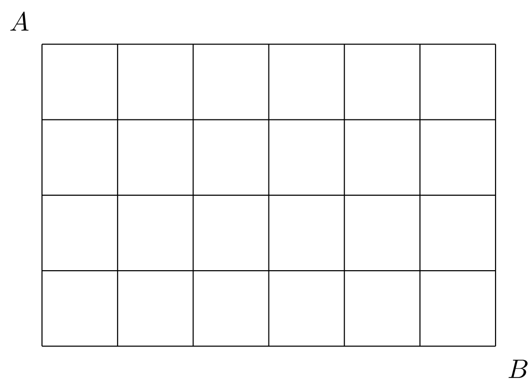
Problem 4. How many ways are there to seat n people at a round table?

Problem 5. Given a standard deck of 52 playing cards, how many hands of 5 cards can one form?

Subset Rule. The number of k-element subsets of an n-element set is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Problem 6. How many shortest routes are there from A to B in the grid-like city plan below?



Problem 7. How many ways can you pick 20 donuts from a selection of 5 flavors?

Problem 8. How many non-negative integer solutions does the following equation have?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 20$$

Problem 9. How many outcomes are possible when we roll 5 dice that are differently colored? How many outcomes are possible when we roll 5 identical white dice?