Lecture 29: More Counting

Date: November 13, 2019.

Permutations. Number of ways of ordering r objects out of a set containing n objects is

$$P(n,r) = n \times (n-1) \times \cdots \times (n-(r-1)) = \frac{n!}{(n-r)!}.$$

Subset Rule. The number of k-element subsets of an n-element set is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Problem 1. How many ways can you pick 20 donuts from a selection of 5 flavors?

Problem 2. How many non-negative integer solutions does the following equation have?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 20$$

Problem 3. How many outcomes are possible when we roll 5 dice that are differently colored? How many outcomes are possible when we roll 5 identical white dice?

Problem 4. We want to form 4 baseball teams, red, blue, green, and yellow, from 36 players. In how many ways can this be accomplished?

Problem 5. How many ways are there to rearrange the letters of the word BOOKKEEPER?

Problem 6. What is the coefficient of $x^k y^{n-k}$ in the expansion of $(x+y)^n$?

Problem 7. What is $\sum_{k=0}^{n} \binom{n}{k}$?

Problem 8. What is $\sum_{i \text{ odd}} \binom{n}{i}$? What is $\sum_{i \text{ even}} \binom{n}{i}$?

Problem 9. What is the coefficient of $be^3k^2o^2pr$ in the expansion of $(b+e+k+o+p+r)^{10}$?