Lecture 29: More Counting

Date: November 13, 2019.

**Permutations.** Number of ways of ordering $r$ objects out of a set containing $n$ objects is

$$P(n, r) = n \times (n-1) \times \cdots \times (n-(r-1)) = \frac{n!}{(n-r)!}.$$ 

**Subset Rule.** The number of $k$-element subsets of an $n$-element set is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$ 

**Problem 1.** How many ways can you pick 20 donuts from a selection of 5 flavors?

**Problem 2.** How many non-negative integer solutions does the following equation have?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 20$$

**Problem 3.** How many outcomes are possible when we roll 5 dice that are differently colored?

How many outcomes are possible when we roll 5 identical white dice?

**Problem 4.** We want to form 4 baseball teams, red, blue, green, and yellow, from 36 players. In how many ways can this be accomplished?
**Problem 5.** How many ways are there to rearrange the letters of the word BOOKKEEPER?

**Problem 6.** What is the coefficient of $x^k y^{n-k}$ in the expansion of $(x+y)^n$?

**Problem 7.** What is $\sum_{k=0}^{n} \binom{n}{k}$?

**Problem 8.** What is $\sum_{i \text{ odd}} \binom{n}{i}$? What is $\sum_{i \text{ even}} \binom{n}{i}$?

**Problem 9.** What is the coefficient of $b e^3 k^2 o^2 pr$ in the expansion of $(b + e + k + o + p + r)^{10}$?