Lecture 3: Propositional and First order Logic

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Recap:

- A **proposition** is a statement that is either true or false.
- If P and Q are propositions, then so are $\mathsf{NOT}(P)$, P AND Q, P OR Q, P $\mathsf{IMPLIES}$ Q, and P IFF Q. The meaning of these combined propositions is given using a truth table.
- Two expressions are logically equivalent if they evaluate to the same truth value in **all** situations, i.e., in **every** row of the truth table they take the same value.
- The **contrapostive** of an implication P $\mathsf{IMPLIES}$ Q is $(\mathsf{NOT}(Q))$ $\mathsf{IMPLIES}$ $(\mathsf{NOT}(P))$. The contrapositive is **logically equivalent** to the implication.
- The **converse** of an implication P $\mathsf{IMPLIES}$ Q is Q $\mathsf{IMPLIES}$ P. The converse is **not logically equivalent** to the implication.

Useful Logical Equivalences

$\mathsf{NOT}(\mathsf{NOT}(P)) \equiv P$	$P \mathsf{\ OR\ } Q \equiv Q \mathsf{\ OR\ } P$
$\mathsf{NOT}(P \mathsf{\ OR\ } Q) \equiv (\mathsf{NOT}(P)) \mathsf{\ AND\ } (\mathsf{NOT}(Q))$	$P \mathsf{\ AND\ } Q \equiv Q \mathsf{\ AND\ } P$
$\mathsf{NOT}(P \mathsf{\ AND\ } Q) \equiv (\mathsf{NOT}(P)) \mathsf{\ OR\ } (\mathsf{NOT}(Q))$	$P \mathsf{\ AND\ } (Q \mathsf{\ AND\ } R) \equiv (P \mathsf{\ AND\ } Q) \mathsf{\ AND\ } R$
$\mathsf{NOT}(P \mathsf{\ IMPLIES\ } Q) \equiv P \mathsf{\ AND\ } (\mathsf{NOT}(Q))$	$P \mathsf{\ OR\ } (Q \mathsf{\ OR\ } R) \equiv (P \mathsf{\ OR\ } Q) \mathsf{\ OR\ } R$
$P \mathsf{\ OR\ } (Q \mathsf{\ AND\ } R) \equiv (P \mathsf{\ OR\ } Q) \mathsf{\ AND\ } (P \mathsf{\ OR\ } R)$	$P \mathsf{\ AND\ } (Q \mathsf{\ OR\ } R) \equiv (P \mathsf{\ AND\ } Q) \mathsf{\ OR\ } (P \mathsf{\ AND\ } R)$

Question 1.

1. What is T OR P? Is it (a) T, (b) F, or (c) P?
2. What is F OR P? Is it (a) T, (b) F, or (c) P?
3. What is T AND P? Is it (a) T, (b) F, or (c) P?
4. What is F AND P? Is it (a) T, (b) F, or (c) P?
5. Are P $\mathsf{IMPLIES}$ $(Q$ $\mathsf{IMPLIES}$ $R)$ and $(P$ $\mathsf{IMPLIES}$ $Q)$ $\mathsf{IMPLIES}$ R equivalent?

Definition 1. A formula is **valid** if it is always true, no matter what truth values are assigned to the variables.

A formula is **satisfiable** if there is some truth assignment to the variables under which it evaluates to true.

Question 2. For each of the following expressions, determine if it is satisfiable or valid.
(a) P OR Q, (b) P OR $(\mathsf{NOT}(P))$, (c) P AND $(\mathsf{NOT}(P))$.

Question 3. Is the following statement true? If an expression φ is satisfiable then φ is valid.

Question 4. Suppose φ is valid. Then $\mathsf{NOT}(\varphi)$ is (a) satisfiable, (b) valid, (c) not satisfiable.

Definition 2. A **predicate** is a proposition that depends on the value of variables.

Universal Quantification	**Existential Quantification**
$\forall x \in \mathbb{Z}.\ x^2 \geq 0$	$\exists x \in \mathbb{Z}.\ x^2 - 4 = 0$