Lecture 31: Principle of Inclusion-Exclusion and Combinatorial Proofs

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Principle of Inclusion-Exclusion.

- For any sets A, B, $|A \cup B| = |A| + |B| - |A \cap B|$
- For any sets A, B, C, $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.
- More generally, for any sets $S_1, S_2, \ldots S_n$,

$$|\bigcup_{i=1}^{n} S_i| = \sum_{\emptyset \neq I \subseteq \{1,\ldots n\}} (-1)^{|I|+1} |\bigcap_{i \in I} S_i|$$

Problem 1. In how many permutations of the set $\{0, 1, 2, \ldots 9\}$ do either 4 and 2, 0 and 4 or 6 and 0 appear consecutively?

Problem 2. Suppose we have 4 distinct letters to be placed in 4 different pre-addressed envelopes. How many ways can we place letters in envelopes so that no letter is placed in the right envelope?

Problem 3. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Problem 4. $2^n = \sum_{i=0}^{n} \binom{n}{i}$

Problem 5. $\sum_{i=k}^{n} \binom{i}{k} = \binom{n+1}{k+1}$