Lecture 18: (Undirected) Graphs

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Definitions

Graph. A (simple, undirected) graph G consists of a non-empty set of vertices $V(G)$ and a set of edges $E(G)$. Each edge $e \in E(G)$ is a two element subset of $V(G)$, i.e., it is of the form $\{u, v\}$ where $u \neq v$.

- Vertices u and v are said to be **end points** of edge $\{u, v\}$.
- Edge $\{u, v\}$ is said to be **incident** to u and v.
- Vertices u and v are said to **adjacent** if $\{u, v\} \in E(G)$.
- The **degree** of vertex v, $\mathsf{deg}(v)$, is the number of edges incident on v.

Example Graphs.

Problem 1. On average, who has more opposite-gender partners: men or women?

Problem 2. Alice and Bob are describing a party they both attended where every person at the party shook hands with exactly 5 other people. However, Alice and Bob disagree on how many people were there at the party. Alice claims there were 126 people, while Bob claims there were 173. Who among Alice and Bob is definitely wrong?

Isomorphism

Definition. An **isomorphism** between graphs G and H is a bijection $f : V(G) \to V(H)$ such that

$$\{u, v\} \in E(G) \text{ IFF } \{f(u), f(v)\} \in E(H).$$

G and H are said to **isomorphic** if there is (some) isomorphism between G and H.

Question 1. Let G and H be isomorphic graphs. For each of the following statements decide if it is necessarily true. (a) $V(G) = V(H)$ (b) $|V(G)| = |V(H)|$ (c) $E(G) = E(H)$ (d) $|E(G)| = |E(H)|$