Lecture 20: Bipartite Graphs, and Coloring

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Special Walks and Tours

Eulerian Tour of G is a closed walk that includes every edge exactly once.

Theorem 1. *A connected graph has an Eulerian tour if and only if every vertex has an even degree.*

Hamiltonian Cycle of G is a cycle that visits every vertex in G exactly once.

Bipartite Graphs

Definition. A graph G is **bipartite** if the set of vertices $V(G)$ can be *partitioned* into sets $L(G)$ and $R(G)$ such that every edge has one endpoint in $L(G)$ and the other endpoint in $R(G)$.

Proposition 2. *Every cycle in a bipartite graph has even length.*

```
read a
read b
if (a > b)
    c = a-b
else c = b-a
d = c
e = d + c
f = e/2
```

Coloring

A k-**coloring** of a graph G is $c : V(G) \to \{1, 2, \dots k\}$ such that for any edge $\{u, v\} \in E(G)$, $c(u) \neq c(v)$.

Chromatic number. The least k such that G has a k-coloring is the **chromatic number** of G. It is denoted as $\chi(G)$.

Theorem 3. *A graph G is bipartite if and only if $\chi(G) = 2$.*

Theorem 4. *Let G be a graph such that for every vertex u, $\mathsf{deg}(u) \leq n$. Then $\chi(G) \leq n + 1$.*