Special Walks and Tours

**Eulerian Tour** of $G$ is a closed walk that includes every edge exactly once.

**Theorem 1.** A connected graph has an Eulerian tour if and only if every vertex has an even degree.

**Hamiltonian Cycle** of $G$ is a cycle that visits every vertex in $G$ exactly once.

Bipartite Graphs

**Definition.** A graph $G$ is **bipartite** if the set of vertices $V(G)$ can be partitioned into sets $L(G)$ and $R(G)$ such that every edge has one endpoint in $L(G)$ and the other endpoint in $R(G)$.

**Proposition 2.** Every cycle in a bipartite graph has even length.
Coloring

A $k$-coloring of a graph $G$ is $c : V(G) \rightarrow \{1, 2, \ldots, k\}$ such that for any edge $\{u, v\} \in E(G)$, $c(u) \neq c(v)$.

Chromatic number. The least $k$ such that $G$ has a $k$-coloring is the chromatic number of $G$. It is denoted as $\chi(G)$.

Theorem 3. A graph $G$ is bipartite if and only if $\chi(G) = 2$.

Theorem 4. Let $G$ be a graph such that for every vertex $u$, $\deg(u) \leq n$. Then $\chi(G) \leq n + 1$. 