Lecture 23: Big Oh

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Big Oh. For $f, g : \mathbb{N} \to \mathbb{N}$, we say that $f = O(g)$ (f is **asymptotically at most as large** as g) iff there is c, k such that for every $n \geq k$, $f(n) \leq cg(n)$.

Problem 1. Show that $\frac{(n-1)n}{2} = O(n^2)$ and $n^2 = O(\frac{(n-1)n}{2})$.

Proposition 1. *For any k, and $a_0, a_1, \ldots a_k$, $\sum_{i=0}^{k} a_i x^i = O(x^k)$.*

Proposition 2. *If $f = O(h)$ and $g = O(h)$ then $f \pm g = O(h)$.*

Common Tips and Pitfalls

Theta Notation. For function $f, g : \mathbb{N} \to \mathbb{N}$, $f = \Theta(g)$ iff $f = O(g)$ and $g = O(f)$.

Little Oh. For functions $f, g : \mathbb{N} \to \mathbb{N}$, we say $f = o(g)$ (f is **asymptotically smaller** than g) iff

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

Problem 2. Show that $n^2 = o(2^n)$.

Big Omega. For functions $f, g : \mathbb{N} \to \mathbb{N}$, we say $f = \Omega(g)$ (f is **asymptotically at least as large** as g) iff there are c, k such that for all $n \geq k$, $f(n) \geq cg(n)$.

Proposition 3. *$f = \Omega(g)$ iff $g = O(f)$.*