CS 173 Discussion 13: Counting II

Date: November 20/21, 2019.

Problem 1. Two people will be said to be *acquaintances* if they have met before, and they will be said to be *strangers* if they have never met before. Prove that in any group of 6 people, there is either a group of 3 people who are mutual acquantainces (i.e., any two in this group of 3 have met before) or there is a group of 3 mutual strangers (i.e., no two in this group of 3 have met before).

Aside: The above result is a special case of a result due to Ramsey that set-off the sub-field within combinatorics called Ramsey Theory that tries to "find ordered regularity among disorder" — find regular sub-structures in any large object. The specific result of Ramsey states that for any ℓ and k, there is a number $R(\ell, k)$ such that in any group of size at least $R(\ell, k)$, there is either a group of size ℓ of mutual acquaintances, or a group of size m of mutual strangers. The special case here says that $6 \geq R(3, 3)$.

Problem 2. How many prime numbers are less than 100? Please solve this problem without actually listing all the prime numbers!

Problem 3. Prove that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$