

---

## CS 173 DISCUSSION 9: GRAPH COLORING AND TREES

Date: October 24/25, 2019.

---

**Problem 1.** CS 173 on semester had 8 discussions section, each of which needed to be run by two to three staff members. The assignment of staff to discussion sections, is as follows.

- R1: Patrick, Tanvi, Belinda
- R2: Patrick, Adithya, Aniket
- R3: Tanvi, Hantian
- R4: Peilun, Adithya, Ruta
- R5: Peilun, Sahand, Aniket
- R6: Sahand, Hantian
- R7: Sahand, Adithya
- R8: Tanvi, Hantian, Aniket

Two discussions cannot be held in the same 50-minute slot if some staff member is assigned to both discussions. The problem is to determine the minimum number of time slots required to complete all the discussions.

1. Recast this problem as a question about coloring vertices of a particular graph. Draw the graph and explain what the vertices, edges, and colors represent.
2. Show a coloring of this graph using the fewest possible colors. Explain why no fewer colors will work. What schedule of discussions does this imply?

**Problem 2.** A *binomial tree* of order  $k$  is a rooted tree defined recursively as follows.

1. A single root vertex is a binomial tree of order 0.
2. A binomial tree of order  $k$  consists of two binomial trees of order  $k - 1$ , with the root of the second is connected as the rightmost child of the root of the first.

The following figure shows binomial trees of order 1, 2, and 3. Below, the vertices labeled “ $B$ ” refer to vertices of the second tree, while vertices labeled “ $A$ ” are those of the first tree. Prove that for any  $k$ , a binomial tree of order  $k$  has  $2^k$  vertices.

