Problem 1. Recall the following recursive definitions of strings and concatenation.

- **Base**: \( \lambda \in A^* \)
- **Constructor**: if \( a \in A, s \in A^* \), then \( \langle a, s \rangle \in A^* \)
- **Constructor**: \( \langle a, s \rangle \cdot t = \langle a, s \cdot t \rangle \)

Recall also that we proved in class that for any string \( s \), \( s \cdot \lambda = s \).

1. Let \( \text{rev}(s) \) denote the *reversal* of string \( s \) (i.e., \( s \) written backwards). Give a recursive definition of \( \text{rev}(s) \).

2. Prove that \( \text{rev}(s \cdot t) = \text{rev}(t) \cdot \text{rev}(s) \). You may assume (or prove it as an exercise at home) that concatenation is *associative*, i.e., for any strings \( r, s, t \), \( (r \cdot s) \cdot t = r \cdot (s \cdot t) \).

Problem 2 (de Bruijn sequences and Graphs). Consider the set of length 3 strings over the binary alphabet \( \{0,1\} \). A *de Bruijn sequence* is binary string where every length 3 binary string appears at least once. For example, we could concatenate each length 3 string in the usual order (000, 001, and so on) to get a string of length \( 3 \times 8 = 24 \) of the form 00001010···.

(a) Find a string of length 10 that is a de Bruijn sequence.

De Bruijn sequences can be found by constructing walks in a special graph called a *de Bruijn graph*. Such a graph \( D \) is defined as follows. The set of vertices \( V(D) = \{ij \mid i, j \in \{0,1\} \} \), and \( E(D) = \{(ij, jk) \mid i, j, k \in \{0,1\} \} \).

(b) Draw the de Bruijn graph.

(c) Explain how any walk that includes every edge in the de Bruijn graph determines a de Bruijn sequence.

(d) Explain how every de Bruijn sequence corresponds to a walk that visits every edge in the graph at least once.

(e) Use the above observations to argue that 10 is the minimum length for a de Bruijn sequence.

De Bruijn sequences and graphs as defined here have been specialized to the case of the binary alphabet (as opposed to a general alphabet) and length 3 strings (as opposed to a general length). See the wikipedia page ([https://en.wikipedia.org/wiki/De_Bruijn_sequence](https://en.wikipedia.org/wiki/De_Bruijn_sequence)) for more information on such graphs and sequences.