CS 173 Discussion 6: Number Theory and Invariant Method

Date: October 3/4, 2019.

Problem 1. A robot named Wall-E wanders around a two-dimensional grid. He starts out at $(0, 0)$ and is allowed to take four different types of steps.

1. $(+2, -1)$
2. $(+1, -2)$
3. $(+1, +1)$
4. $(-3, 0)$

For example, Wall-E might walk as follows.

$$(0,0) \xrightarrow{1} (2,-1) \xrightarrow{3} (3,0) \xrightarrow{2} (4,-2) \xrightarrow{4} (1,-2) \cdots$$

Wall-E's true love, the fashionable and high-powered robot, Eve, awaits in $(0, 2)$.

(a) Describe a state machine model of this problem.

(b) Will Wall-E ever find his true love? If yes, find a path from Wall-E to Eve. If not, use the Invariant Principle to prove no such path exists.

Problem 2. Recall that we showed that for any n, congruence modulo n is an equivalence relation. What is the equivalence class of 8 modulo 3?

Problem 3. Prove or disprove each of the following statements.

1. For any integers d, ℓ, m, and n, if $\ell \equiv m \pmod{n}$ then $d^\ell \equiv d^m \pmod{n}$.
2. For any integers p, q, r, if $gcd(p, q) = 1$ and $\gcd(q, r) = 1$ then $\gcd(p, r) = 1$.
3. For any non-zero integers p, q, r, if $\gcd(p, q) = 1$ and $\gcd(p, qr) = 1$ then $\gcd(p, r) = 1$.