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## CS 173 DISCUSSION 5: INDUCTION

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**Problem 1.** Prove the following formulas using induction.

1.  $\sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
2.  $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$

**Problem 2.** Consider the following induction proof showing for all  $n \geq 0$ ,  $2 + 3 + \dots + n = \frac{n(n+1)}{2}$ .

**Proof:** We use induction. Let  $P(n)$  be the proposition that  $2 + 3 + \dots + n = \frac{n(n+1)}{2}$ .

*Base Case.*  $P(0)$  is true since both sides of the equation of  $P(0)$  are 0, as the sum with no terms is zero.

*Induction Hypothesis.* For  $n > 0$ , assume that  $P(0), P(1), \dots, P(n-1)$  hold.

*Induction Step.* Observe that

$$\begin{aligned} 2 + 3 + \dots + (n-1) + n &= (2 + 3 + \dots + (n-1)) + n \\ &= \frac{(n-1)(n-1+1)}{2} + n \\ &= \frac{n((n-1)+2)}{2} = \frac{n(n+1)}{2} \end{aligned}$$

Therefore by the principle of induction  $\forall n \in \mathbb{N}$ .  $P(n)$  is true.

What is the error in the above proof?