CS 173, Fall 2015
Examlet 13, Part A

NETID:

FIRST:	LAST:

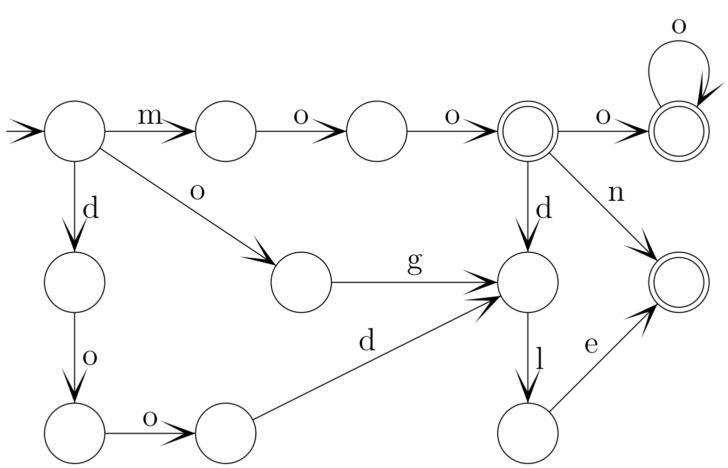
Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(15 points) Recall that a phone lattice is a state diagram representing sequences of letters. Each edge in a phone lattice has a single letter on it. In a "deterministic" state diagram, if you look at any state s and any letter a, there is never more than one edge labelled a leaving state s.

Draw a deterministic phone lattice representing exactly the following set of words, using no more than 15 states and, if you can, no more than 13.

moodle, moon, doodle, ogle
moo, mooo, moooo, ... [i.e. m followed by two or more o's]

Solution:



CS 173, Fall 2015
Examlet 13, Part B

NETID:

FIRST: | **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(5 points) A "red/black tree" is a binary tree, each of whose nodes contains either "red" or "black." Is the set of all red/black trees countable or uncountable? Briefly justify your answer.

Solution: This is countable. For any n, there are only a finite number of distinct binary trees with n nodes. A tree with n nodes can be colored in 2^n ways. So there can only be a finite number of red/black trees of each size. Then the whole set is the union of countably many finite sets, which is countable.

(10 points) Check the (single) box that best characterizes each item.

The set of all intervals $[a, b]$ of the real line. finite ☐ countably infinite ☐ uncountable $\boxed{\surd}$

The set of board configurations for the game of chess. finite $\boxed{\surd}$ countably infinite ☐ uncountable ☐

Every function from $\{1, 2, 3\}$ to the reals has a finite formula. true ☐ false $\boxed{\surd}$ not known ☐

The set of all (finite, unlabelled) graphs, where isomorphic graphs are treated as the same object. finite ☐ countably infinite $\boxed{\surd}$ uncountable ☐

$\mathbb{P}(\mathbb{N})$ finite ☐ countably infinite ☐ uncountable $\boxed{\surd}$

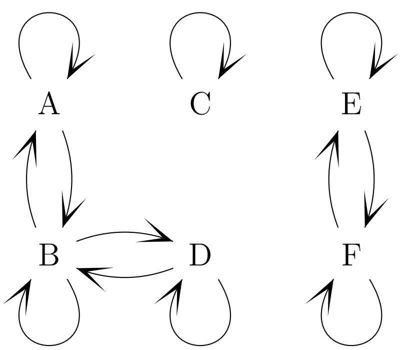
CS 173, Fall 2015
Review, Part A

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: [√] Irreflexive: [ ]

Symmetric: [√] Antisymmetric: [ ]

Transitive: [ ]

(10 points) Check the (single) box that best characterizes each item.

For any positive integers p, q, and k,
if $p \equiv q \pmod{k}$, then $p^2 \equiv q^2 \pmod{k}$

true [√] false [ ]

$\forall x \in \mathbb{R}$, if $\pi = 3$, then $x < 20$.
(π is the familiar constant.)

true [√] false [ ] undefined [ ]

$$\sum_{k=0}^{n-1} 2^k$$

$2^n - 2$ [ ] $2^n - 1$ [√] $2^{n-1} - 1$ [ ] $2^{n+1} - 1$ [ ]

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the _____ of f.

domain [ ] co-domain [ ]
image [√]

$f : \mathbb{R} \to \mathbb{Z}$,
$f(x) = x$

one-to-one [ ] not one-to-one [ ] not a function [√]

CS 173, Fall 2015
Review, Part B

NETID:

FIRST:	**LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

(5 points) Is the graph C_{10} bipartite? Briefly justify your answer.

Solution: Yes, it is bipartite. As you walk around the cycle, assign nodes to the two subsets in an alternating manner.

(10 points) Check the (single) box that best characterizes each item.

Suppose I want to estimate $\frac{103}{20}$. 3 is ______

an upper bound	☐	an exact answer	☐
a lower bound	☑ $\surd$	not a bound on	☐

The chromatic number of the 3-dimensional hypercube Q_3

1	☐	2	☑ $\surd$	3	☐	4	☐

Total number of leaves in a 3-ary tree of height h

3^h	☐	$\leq 3^h$	☑ $\surd$
$\frac{1}{2}(3^{h+1} - 1)$	☐	$3^{h+1} - 1$	☐

$T(1) = d$
$T(n) = 2T(n-1) + c$

$\Theta(n)$	☐	$\Theta(n^2)$	☐	$\Theta(n \log n)$	☐	$\Theta(2^n)$	☑ $\surd$

The running time of mergesort is $O(n^3)$.

True	☑ $\surd$	False	☐