CS 173, Fall 2015
Examlet 9, Part B

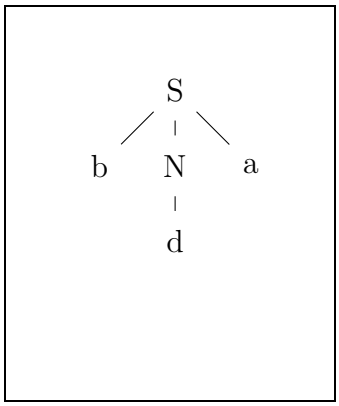
NETID:

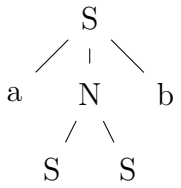
FIRST: | **LAST:**

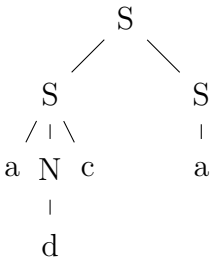
Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

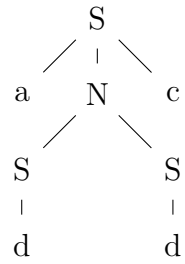
1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$S \rightarrow \;\; b\ N\ a \;\mid\; a\ N\ c \;\mid\; a$
$N \rightarrow \;\; S\ S \;\mid\; d$









2. (4 points) Check the (single) box that best characterizes each item.

The level of the root node in a tree of height h.

0 [√] 1 [ ] $h-1$ [ ] h [ ] $h+1$ [ ]

$\sum_{k=0}^{n+1} 2^k$ $\quad 2^{n+1}+1$ [ ] $\quad 2^{n+2}-1$ [√] $\quad 2^{n+2}-2$ [ ] $\quad 2^{n+1}-1$ [ ]

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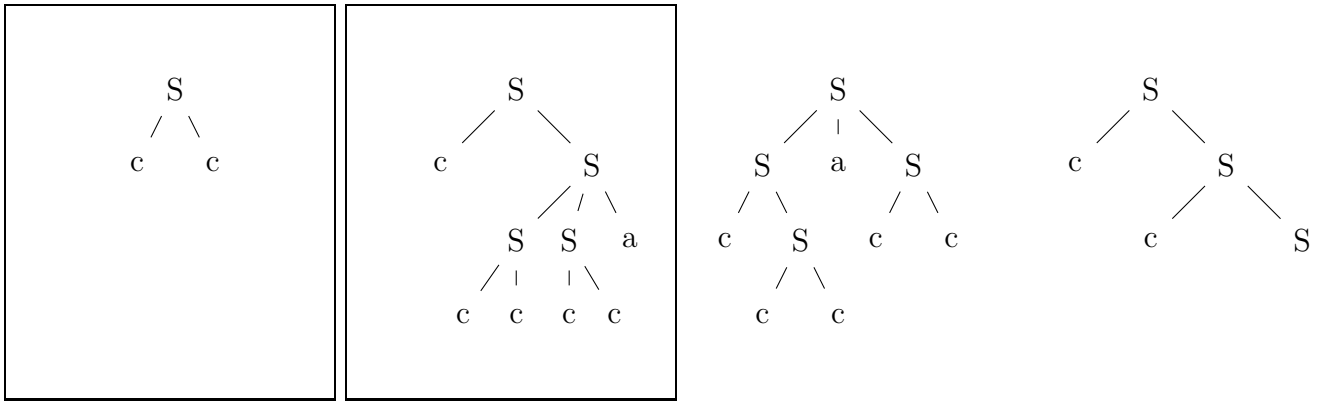
1. (8 points) Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \rightarrow S\ S\ a \mid c\ S \mid c\ c$$



2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a full complete binary tree of height h

$\geq 2^h$	☐	$= 2^{h+1} - 1$	☑
$\leq 2^{h+1} - 1$	☐	$\geq 2^{h+1} - 1$	☐

Total number of leaves in a 3-ary tree of height h

3^h	☐	$\leq 3^h$	☑
$\frac{1}{2}(3^{h+1} - 1)$	☐	$3^{h+1} - 1$	☐

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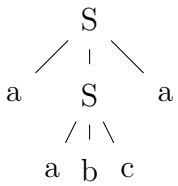
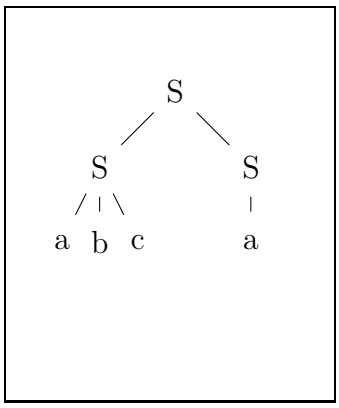
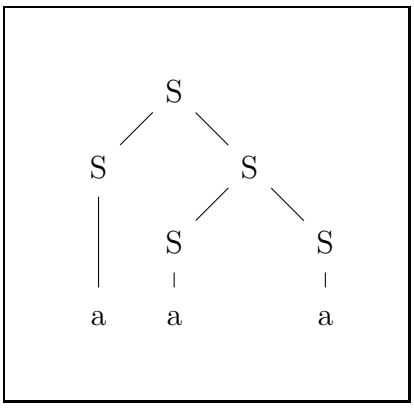
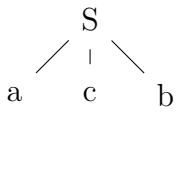
1. (8 points) Here is a grammar with start symbol S and terminals symbols a,b, and c. Circle the trees that match the grammar.

$$S \rightarrow S\ S \mid a\ b\ c \mid a$$









2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=0}^{n-1} 2^k$ $\quad 2^n - 2$ ☐ $\quad 2^n - 1$ ☑ $\quad 2^{n-1} - 1$ ☐ $\quad 2^{n+1} - 1$ ☐

The number of nodes in a binary tree of height h

$\geq 2^h$ ☐ $\quad = 2^{h+1} - 1$ ☐

$\leq 2^{h+1} - 1$ ☑ $\quad \geq 2^{h+1} - 1$ ☐

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1. (8 points) Consider the following grammar G

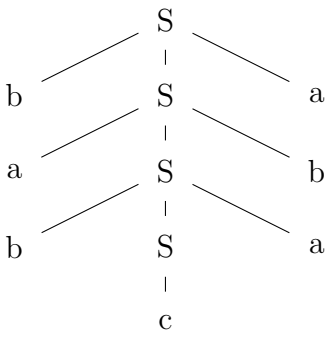
   $S \to b\ S\ a \mid a\ S\ b \mid c$

   S is the only start symbol. The terminal symbols are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $b\ a\ b\ c\ a\ b\ a$ $\qquad\qquad$ $b\ b\ a\ c\ b\ a\ b$

   Solution:



This is impossible. The grammar produces a's and b's at the same time, so its strings always have the same number of a's and b's.

2. (4 points) Check the (single) box that best characterizes each item.

   A binary tree of height h has at least $2^h - 1$ vertices (nodes). true ☐ false ☑

   A tree with n nodes has
   n edges ☐ $n-1$ edges ☑ $\leq n$ edges ☐
   $n/2$ edges ☐ $\log n$ edges ☐

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1. (8 points) Consider the following grammar G

   $S \rightarrow b\ S\ a \ \mid\ b\ S\ b \ \mid\ c$

   S is the only start symbol. The terminal symbols are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $b\ a\ b\ c\ b\ b\ b$ $\qquad\qquad$ $b\ b\ c\ a\ b\ a\ b$

   Solution:

   This is impossible. In strings produced by G, all a's occur after the c.

   This is impossible. All strings produced by G have the (single) c in the exact middle of the string.

2. (4 points) Check the (single) box that best characterizes each item.

   Number of nodes at level k in a full complete binary tree.

2^k	√	$2^k - 1$	
$2^{k+1} - 1$		2^{k-1}	

   The chromatic number of a full 3-ary tree

1		2		≤ 2	√
3		≤ 3		can't tell	

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1. (8 points) Consider the following grammar G

   $S \rightarrow\ a\ S\ \mid\ a\ N$

   $N \rightarrow\ N\ N\ \mid\ b\ c\ \mid\ c\ c$

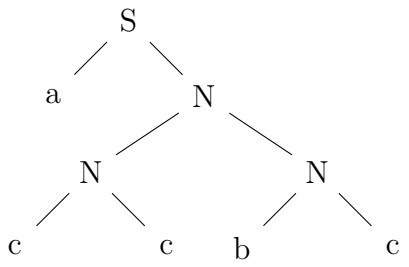
   S is the only start symbol. The terminal symbol are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $a\ b\ c\ c\ c\ a$ $\qquad\qquad$ $a\ c\ c\ b\ c$

   Solution:

   Impossible because this grammar can generate a's only at the start of the string, before all the b's and c's.



2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n+1} 2^k$ $\qquad$ $2^{n+1}+1$ ☐ $\qquad$ $2^{n+2}-1$ ☐ $\qquad$ $2^{n+2}-2$ ☑ $\qquad$ 2^n-2 ☐

A full m-ary tree with i internal nodes has $mi+1$ nodes total. $\qquad$ always ☑ $\qquad$ sometimes ☐ $\qquad$ never ☐