# CS 173, Fall 2015
# Examlet 9, Part A

**NETID:**

| **FIRST:** | **LAST:** |
|---|---|

**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(18 points) If T is a binary tree with root R, then Mass(T) is defined to be

- 0 if R is a leaf
- m if R has one child subtree, with Mass m
- 1+m if R has two child subtrees, both with Mass m
- otherwise, the maximum Mass of R's two child subtrees.

Use (strong) induction to prove that a binary tree T with Mass(T)=p has at least $2^p$ leaves

The induction variable is named ______ and it is the ____________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(18 points) A Western tree is a full binary tree whose nodes contain integers such that

- Every leaf contains the value 0.
- The value $v(X)$ in a node X is (strictly) larger than the values in X's children.

Use (strong) induction to prove that the value in the root of a Western tree is larger than the value in any other node of the tree.

The induction variable is named ______ and it is the ____________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(18 points) A Raven tree is a binary tree in which each node X contains an integer label $v(X)$ such that

- If $X$ is a leaf, $v(X)$ is 7, 23, or 31.
- If $X$ has one child $Y$, then $v(X) = v(Y) + 7$.
- If $X$ has two children $Y$ and $Z$, then $v(X) = v(Y)v(Z)$.

Use strong induction to prove that the value in the root of a Raven tree is always positive.

The induction variable is named ______ and it is the ________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(18 points) Monster trees are binary trees whose nodes are labelled with strings, such that

- Each leaf node has label `left`, `right`, or `back`
- If a node has one child, it has label $\alpha$`x` where where $\alpha$ is the child's label. E.g. if the child has label `left` then the parent has `leftx`.
- If a node has two children, it contains $\alpha\beta$ where $\alpha$ and $\beta$ are the child labels. E.g. if the children have labels `right` and `back`, then the parent has label `rightback`.

Let S(n) be the length of the label on node n. Let L(n) be the number of leaves in the subtree rooted at n. Use (strong) induction to prove that $S(n) \geq 4L(n)$ if $n$ is the root node of any Monster tree.

The induction variable is named ______ and it is the __________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(18 points) A Tippy tree is a full binary tree in which each node is colored orange or blue, such that:

- If $v$ is a leaf node, then $v$ is colored orange.
- If $v$ has two children of the same color, then $v$ is colored blue.
- If $v$ has two children of different colors, then $v$ is colored orange.

Use (strong) induction to show that the root of an Tippy tree is blue if and only if the tree has an even number of leaves. You may assume basic divisibility facts e.g. the sum of two odd numbers is even.

The induction variable is named ______ and it is the ____________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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**Discussion:** **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

(18 points) Here is a grammar $G$, with start symbol $S$ and terminal symbols $a$ and $b$.

$$S \rightarrow a\ S\ b \ |\ b\ S\ a \ |\ S\ S \ |\ a\ b$$

Use (strong) induction to prove that any tree matching (aka generated by) grammar $G$ has equal numbers of a's and b's. Use $A(T)$ and $B(T)$ as shorthand for the number of a's and b's in a tree $T$.

The induction variable is named ______ and it is the __________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: