CS 173, Fall 2015
Examlet 8, Part B

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(10 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= 3 \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + \sum_{p=0}^{k-1} n2^p$$

Finish finding the closed form for $g(n)$ assuming that n is a power of 2. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

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1. (8 points) Suppose we have a function g defined (for n a power of 3) by

$$\begin{aligned} g(9) &= 5 \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 27 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 3^k g(n/3^k) + kn$$

Finish finding the closed form for g. Show your work and simplify your answer.

2. (2 points) Check the (single) box that best characterizes each item.

The number of nodes in the 4-dimensional hypercube Q_4 — 4 ☐ 16 ☐ 32 ☐ 64 ☐

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1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer.

2. (2 points) Suppose that $f : \mathbb{N} \rightarrow \mathbb{N}$ is such that $f(n) = n!$. Give a recursive definition of f

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(10 points) Suppose we have a function F defined (for n a power of 3) by

$$\begin{aligned} F(1) &= 5 \\ F(n) &= 3F(n/3) + 7 \text{ for } n \geq 3 \end{aligned}$$

Your partner has already figured out that

$$F(n) = 3^k F(n/3^k) + 7 \sum_{p=0}^{k-1} 3^p$$

Finish finding the closed form for F. Show your work and simplify your answer. Recall the following useful closed form (for $r \neq 1$): $\sum_{k=0}^{n} r^k = \frac{r^{n+1} - 1}{r - 1}$

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1. (8 points) Suppose we have a function f defined by

$$\begin{aligned} f(0) &= f(1) = 3 \\ f(n) &= 5f(n-2) + d, \text{ for } n \geq 2 \end{aligned}$$

where d is a constant. Express $f(n)$ in terms of $f(n-6)$ (where $n \geq 6$). Show your work and simplify your answer.

2. (2 points) Suppose that G_0 is the graph consisting of a single vertex. Also suppose that the graph G_n consists of a copy of G_{n-1} plus an extra vertex v and edges joining v to each vertex in G_{n-1}. Give a clear picture or precise description of G_4.

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(10 points) Suppose we have a function g defined (for n a power of 4) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 2g(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 2^k g(n/4^k) + n \sum_{p=0}^{k-1} \frac{1}{2^p}$$

Finish finding the closed form for $f(n)$ assuming that n is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.