CS 173, Fall 2015
Examlet 8, Part A

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(20 points) Use (strong) induction to prove that $a - b$ divides $a^n - b^n$, for any integers a and b and any natural number n.

Hint: $(a^n - b^n)(a + b) = (a^{n+1} - b^{n+1}) + ab(a^{n-1} - b^{n-1})$, for any real numbers a and b.

Let a and b be integers.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Recall that the hypercube Q_1 consists of two nodes joined by an edge and that Q_n consists of two copies of Q_{n-1} plus edges connecting corresponding nodes. Use (strong) induction to show Q_n has chromatic number 2 for any natural number $n \geq 1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Use (strong) induction to prove that $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is an integer for all natural numbers n

Hint: $(a^n - b^n)(a+b) = (a^{n+1} - b^{n+1}) + ab(a^{n-1} - b^{n-1})$, for any real numbers a and b.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) (20 points) Suppose that $f : \mathbb{N} \to \mathbb{Z}$ is defined by

$f(0) = 2 \qquad f(1) = 5 \qquad f(2) = 15$

$f(n) = 6f(n-1) - 11f(n-2) + 6f(n-3)$, for all $n \geq 2$

Use (strong) induction to prove that $f(n) = 1 - 2^n + 2 \cdot 3^n$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Recall that the hypercube Q_2 is a 4-cycle, and that Q_n consists of two copies of Q_{n-1} plus edges connecting corresponding nodes. A *Hamiltonian cycle* is a cycle that visits each node exactly once, except obviously for when it returns to the starting node at the end. Use (strong) induction to show Q_n has Hamiltonian cycle for any natural number $n \geq 2$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Let function $f : \mathbb{N} \to \mathbb{Z}$ be defined by

$f(0) = 3$

$f(1) = 9$

$f(n) = f(n-1) + 2f(n-2)$, for $n \geq 2$

Use (strong) induction to prove that $f(n) = 4 \cdot 2^n + (-1)^{n-1}$ for any natural number n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: